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Oxford Cambridge and RSA

The assessment of mathematical reasoning and problem solving in the new GCSE (9-1) Mathematics

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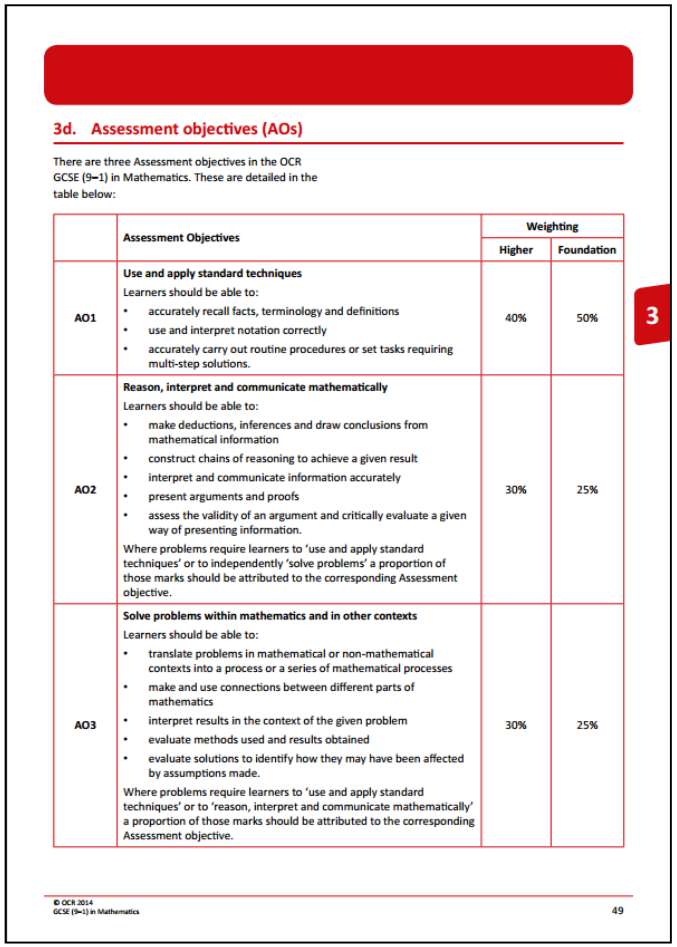
Assessment Objectives

- For assessment purposes, mathematical reasoning and problem solving requirements are defined within a qualification's **Assessment Objectives (AOs)**.

- In GCSE (9-1) Mathematics:
  - AOs are set by DfE
  - have to be adhered to by all awarding bodies' qualifications.



3d. Assessment objectives (AOs)

There are three Assessment objectives in the OCR GCSE (9–1) in Mathematics. These are detailed in the table below:

	Assessment Objectives	Weighting	
		Higher	Foundation
AO1	**Use and apply standard techniques** Learners should be able to: • accurately recall facts, terminology and definitions • use and interpret notation correctly • accurately carry out routine procedures or set tasks requiring multi-step solutions.	40%	50%
AO2	**Reason, interpret and communicate mathematically** Learners should be able to: • make deductions, inferences and draw conclusions from mathematical information • construct chains of reasoning to achieve a given result • interpret and communicate information accurately • present arguments and proofs • assess the validity of an argument and critically evaluate a given way of presenting information. Where problems require learners to 'use and apply standard techniques' or to independently 'solve problems' a proportion of those marks should be attributed to the corresponding Assessment objective.	30%	25%
AO3	**Solve problems within mathematics and in other contexts** Learners should be able to: • translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes • make and use connections between different parts of mathematics • interpret results in the context of the given problem • evaluate methods used and results obtained • evaluate solutions to identify how they may have been affected by assumptions made. Where problems require learners to 'use and apply standard techniques' or to 'reason, interpret and communicate mathematically' a proportion of those marks should be attributed to the corresponding Assessment objective.	30%	25%

3

Legacy GCSE maths Assessment Objectives

- In the legacy GCSE qualifications, no mention was made of reasoning.
- Problem solving was included, but with no further breakdown specified.

Candidates are expected to demonstrate their ability to:

	Assessment Objectives	Weighting (%)
AO1	Recall and use their knowledge of the prescribed content	45-55
AO2	Select and apply mathematical methods in a range of contexts	25-35
AO3	Interpret and analyse problems and generate strategies to solve them	15-25

GCSE (9-1) maths Assessment Objectives

- In GCSE (9-1) Maths, using and applying standard techniques is defined by AO1.

	Assessment Objectives	Weighting	
		Higher	Foundation
AO1	**Use and apply standard techniques** Learners should be able to: • accurately recall facts, terminology and definitions • use and interpret notation correctly • accurately carry out routine procedures or set tasks requiring multi-step solutions.	40%	50%

GCSE (9-1) maths Assessment Objectives

- In GCSE (9-1) Maths, reasoning, communicating and interpreting are defined by AO2.

		Weighting	
		Higher	Foundation
AO2	**Reason, interpret and communicate mathematically** Learners should be able to: • make deductions, inferences and draw conclusions from mathematical information • construct chains of reasoning to achieve a given result • interpret and communicate information accurately • present arguments and proofs • assess the validity of an argument and critically evaluate a given way of presenting information. Where problems require learners to 'use and apply standard techniques' or to independently 'solve problems' a proportion of those marks should be attributed to the corresponding Assessment objective.	30%	25%

GCSE (9-1) maths Assessment Objectives

- In GCSE (9-1) Maths, problem solving requirements are defined by AO3.

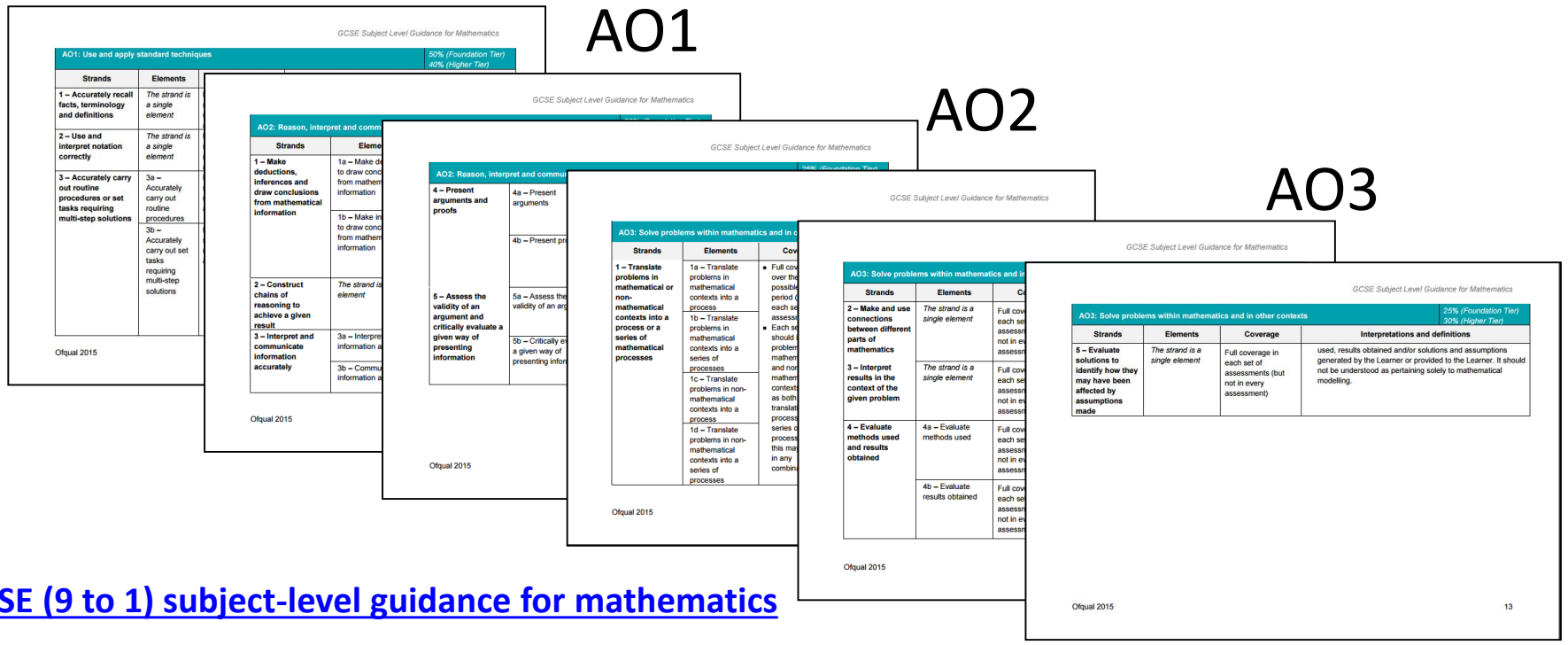
		Weighting	
		Higher	Foundation
AO3	**Solve problems within mathematics and in other contexts** Learners should be able to: • translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes • make and use connections between different parts of mathematics • interpret results in the context of the given problem • evaluate methods used and results obtained • evaluate solutions to identify how they may have been affected by assumptions made. Where problems require learners to 'use and apply standard techniques' or to 'reason, interpret and communicate mathematically' a proportion of those marks should be attributed to the corresponding Assessment objective.	30%	25%

GCSE (9-1) maths Assessment Objectives

- Assessment Objectives are broken down into strands, which are then further broken down into elements.
  - Every element (although not all AO3.1 elements) must be targeted in every assessment series.
- Every mark in every question now requires allocating to an AO element.
  - 'Where problems require learners to *'use and apply standard techniques'* or to *'reason, interpret and communicate mathematically'* a proportion of those marks should be attributed to the corresponding Assessment objective.'

GCSE (9-1) maths Assessment Objectives

- Detailed requirements (e.g. definitions, weightings) are published by Ofqual.



[GCSE (9 to 1) subject-level guidance for mathematics]

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Reasoning in GCSE (9-1) maths

AO2: Reason, interpret and communicate mathematically			25% (Foundation Tier) 30% (Higher Tier)
Strands	**Elements**	**Coverage**	**Interpretations and definitions**
1 – Make deductions, inferences and draw conclusions from mathematical information	1a – Make deductions to draw conclusions from mathematical information	Full coverage in each set of assessments (but not in every assessment)	▪ Strands 1/2/3/4 (but not strand 5) operate on a continuum – they all relate to reasoning, but increase in their level of sophistication; they also vary in terms of whether the Learner is working towards a provided outcome or generating this. ▪ **Deduction** – a process of reasoning from absolutes to results that must be correct ▪ **Inference** – a process of reasoning from more relative or partial evidence to results that are likely to be correct. ▪ **Conclusion** – the result obtained either from a deduction or from an inference.
	1b – Make inferences to draw conclusions from mathematical information	Full coverage in each set of assessments (but not in every assessment)	
2 – Construct chains of reasoning to achieve a given result	*The strand is a single element*	Full coverage in each set of assessments (but not in every assessment)	▪ **Given result** – an outcome that is provided to the Learner; a specific answer required by the task.
3 – Interpret and communicate information accurately	3a – Interpret information accurately	Full coverage in each set of assessments (but not in every assessment)	▪ **Interpret** – working with information in a way that extends beyond what it conveys explicitly.
	3b – Communicate information accurately	Full coverage in each set of assessments (but not in every assessment)	▪ **Communicate** – presenting information in a way that may involve taking something and representing it differently.

Reasoning in GCSE (9-1) maths

AO2: Reason, interpret and communicate mathematically			*25% (Foundation Tier)* *30% (Higher Tier)*
4 – Present arguments and proofs	4a – Present arguments	Full coverage in each set of assessments (but not in every assessment)	▪ **Argument** – a formal, comprehensive and logical account – but with a degree of relativity, such that it may vary in accuracy at different points.
	4b – Present proofs	Full coverage in each set of assessments (but not in every assessment)	▪ **Proof** – a formal, comprehensive and logical account – but also with a degree of absoluteness and incontrovertibility, such that it would be the case at any point; based on the subject content, this will be a requirement at Higher Tier only.
5 – Assess the validity of an argument and critically evaluate a given way of presenting information	5a – Assess the validity of an argument	Full coverage in each set of assessments (but not in every assessment)	▪ Although they are related to each other elements 5a and 5b are distinct.
	5b – Critically evaluate a given way of presenting information	Full coverage in each set of assessments (but not in every assessment)	

Foundation tier – AO2.1

3 Peter says

The sum of an odd number and an even number is even.

The example $3 + 4 = 7$ shows that Peter is **not** correct.

Write an example to show that each of these statements is **not** correct.

(a) The sum of two prime numbers is always odd.

..... **[1]**

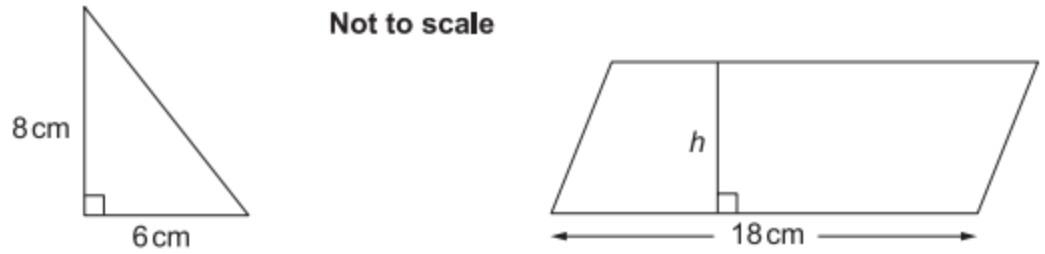
(b) Squaring a whole number always results in an even number.

..... **[1]**

3	**(a)**		Any two odd primes added correctly	**1** **1 AO2.1a**	e.g. $3 + 5 = 8$	
	(b)		An odd integer squared with correct result	**1** **1 AO2.1a**	e.g. $5^2 = 25$	

Foundation tier – AO2.2

(b) The area of the parallelogram is **three** times the area of the triangle.



Show that the perpendicular height h of the parallelogram is 4 cm. **[4]**

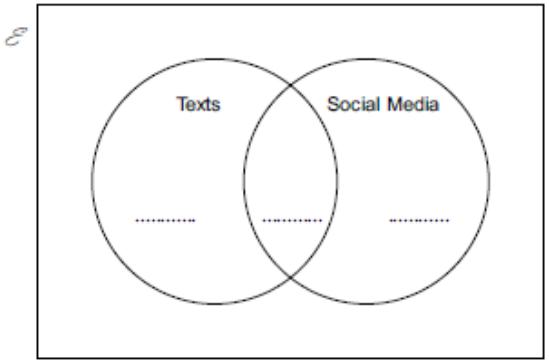
Question	Answer	Marks	Part marks and guidance	
(b)	Correct working leading to 4 cm	4 1 AO1.3b 2 AO2.2 1 AO2.4a	**B1** for area of triangle is 24 **B1** for *their* '24' × 3 **B1** for *their* '72' ÷ 18 or area of parallelogram = 18h	

Foundation tier – AO2.1/AO2.3

50 students were asked in a survey whether they use texts or social media.

- 20 students said they only use texts.
- 8 students said they only use social media.
- 17 students said they use both texts and social media.

(a) Put this information on the Venn diagram.



(b) How many of the students in the survey do not use texts or social media?

(b) ..... **[2]**

(c) One of the students in the survey is chosen at random.

What is the probability that this student uses texts?

New content

Venn Diagrams (OCR will not include Venn diagram notation, e.g. ∩, ∪)

9	**(a)**		20, 8 and 17 in appropriate positions on Venn diagram	**1** 1AO2.3b	
	(b)		5	**2** 1AO2.1a 1AO2.3a	**M1** for 50 – (20 + 17 + 8) **oe**
	(c)		$\frac{37}{50}$	**2** 1AO2.3a 1AO3.3	**M1** for [20 + 17] = 37 **seen**

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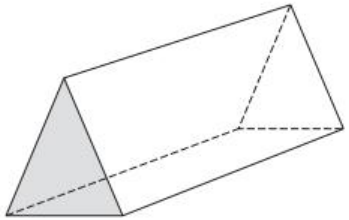
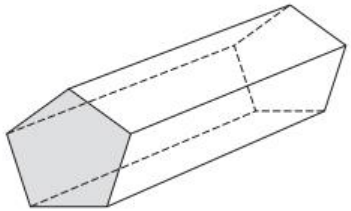
Foundation tier – AO2.1/AO2.3

9 These prisms have different shapes as end faces.



Triangle



Pentagon

(a) Complete this table.

Shape of end face	Number of faces	Number of edges	Number of vertices
Triangle (3 sides)	5	9	6
Rectangle (4 sides)	.....	.....	8
Pentagon (5 sides)	.....	15	10
Hexagon (6 sides)	8	18	.....

[2]

(b) How many edges and vertices does a prism with a 100-sided end face have?

(b) edges .....

vertices .....

[2]

Foundation tier – AO2.1/AO2.3

(c) *F* is the number of faces in a prism.
N is the number of sides of its end face.

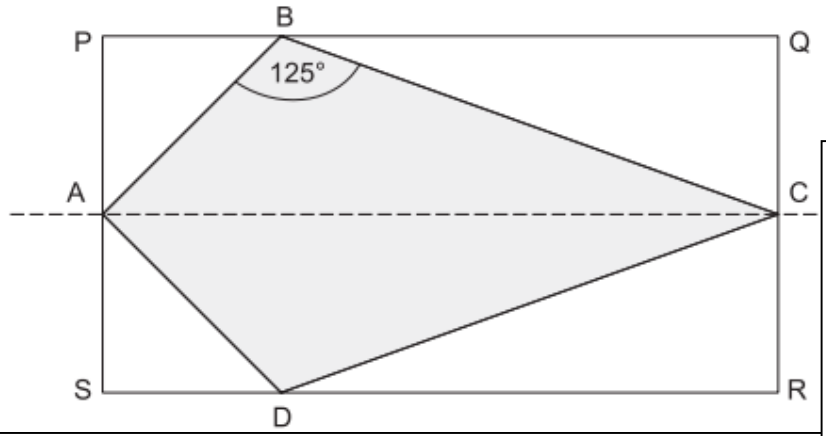
Write down a formula connecting *F* and *N*.

(c) ..... **[2]**

9	**(a)**		<table><tr><th>Prism</th><th>Number of faces</th><th>Number of edges</th><th>Number of vertices</th></tr><tr><td>Triangular (3 sides)</td><td>5</td><td>9</td><td>6</td></tr><tr><td>Rectangular (4 sides)</td><td>**6**</td><td>**12**</td><td>8</td></tr><tr><td>Pentagonal (5 sides)</td><td>**7**</td><td>15</td><td>10</td></tr><tr><td>Hexagonal (6 sides)</td><td>8</td><td>18</td><td>**12**</td></tr></table>	**2** **1 AO1.1** **1 AO2.1a**	**B1** for 2 correct	
	(b)		300 (edges) 200 (vertices)	**1** **1** **2 AO2.1a**		
	(c)		$F = N + 2$ **oe**	**2** **1 AO2.3a** **1 AO2.3b**	**B1** for $N + 2$ (without a subject)	Condone for **B1** a correct word formula

Foundation tier – AO2.4

PQRS is a rectangle.
A, B, C and D are points on SP, PQ, QR and RS respectively.
AC is the line of symmetry for the diagram.



Not to scale

(a) Angle ABC = 125°.

Write down the size of angle ADC.

(a) Angle ADC = ... ° **[1]**

(b) AP is the same length as PB.

Work out the size of angle BCD.
Show your reasoning clearly.

(b) Angle BCD = ... ° **[4]**

Foundation tier – AO2.4

(a) Angle ABC = 125°.

Write down the size of angle ADC.

(a) Angle ADC = ... ° **[1]**

(b) AP is the same length as PB.

Work out the size of angle BCD.
Show your reasoning clearly.

(b) Angle BCD = ... ° **[4]**

7	**(a)**		125	**1** **1 AO1.2**		
	(b)		20	**4** **2 AO2.1a** **2 AO2.4a**	**B1** for PAB = SAD = 45 **B1** for BAD = 90 **M1** for 360 – (*their* '125' + *their* '90' + 125)	May be seen on diagram

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Higher tier – AO2.4

(a) Give **one** reason why 0 is an even number.

.....

..... **[1]**

(b) The lengths of the sides of a **right-angled** triangle are all integers.

Prove that if the lengths of the two shortest sides are even, then the length of the third side must also be even.

.....

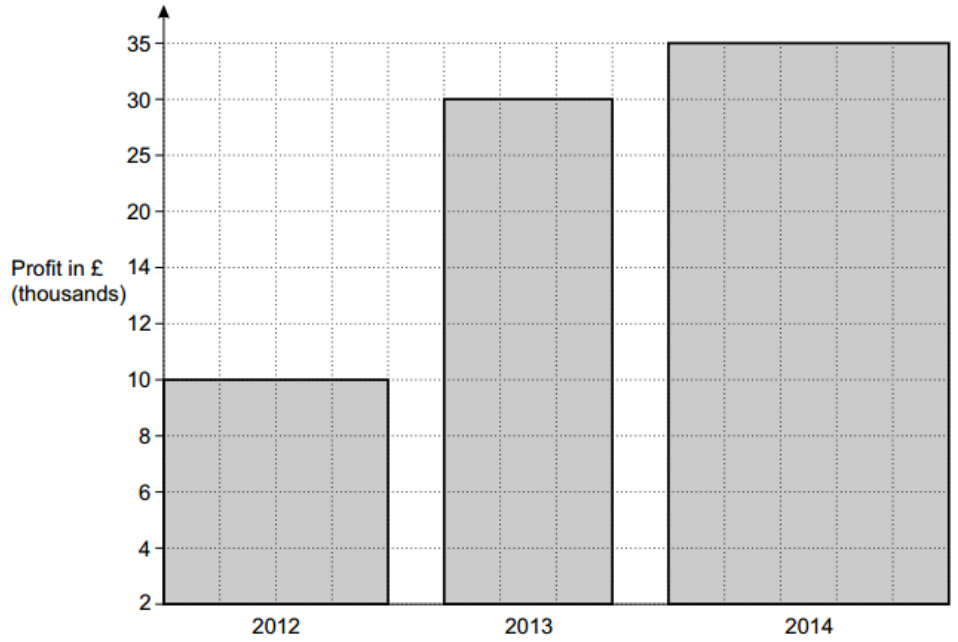
.....

..... **[3]**

11	**(a)**		Any correct reason	**1** **1 AO2.4a**		Exemplar responses: -1 and 1 both odd and either side of 0 Or can be divided by 2 exactly Or numbers that end in 0 are even
	(b)		e.g. $a^2 + b^2 = c^2$ $a = 2x$ and $b = 2y$ implies $c^2 = 4x^2 + 4y^2$ So c is even	**3** **1 AO2.1a** **1 AO2.4b** **1 AO3.2**	**B1** for use of Pythagoras' theorem **M1** for even × even = even **soi**	

Foundation tier – AO2.5

6 This chart shows a firm's profit for each of 3 years.

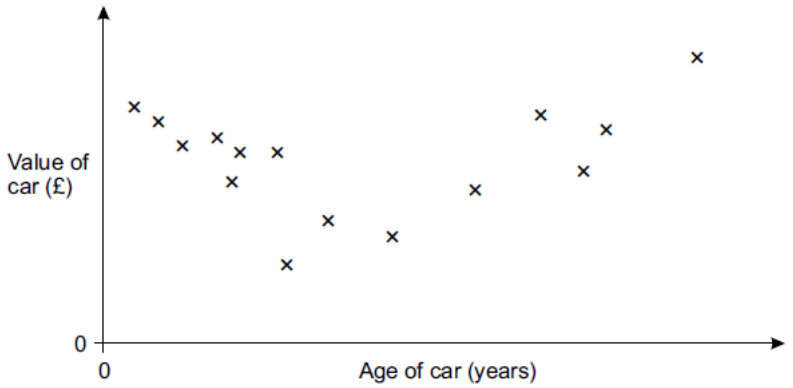


Give **two** reasons why the chart is misleading.

Question			Answer	Marks	Part marks and guidance	
6			Two from: Unequal width bars Frequency/profit scale not linear Vertical axis doesn't start at 0	2 2AO2.5b	**B1** for one reason	

Higher tier – AO2.5

7 This scatter graph shows the values of 15 sports cars plotted against their ages.



(a) **(i)** Lewis thinks that there is **no correlation** between the ages and values of these cars.

Is Lewis correct?
Give a reason for your answer.

.....

..... **[2]**

(ii) Sebastian thinks that there is a **relationship** between the ages and values of these cars.

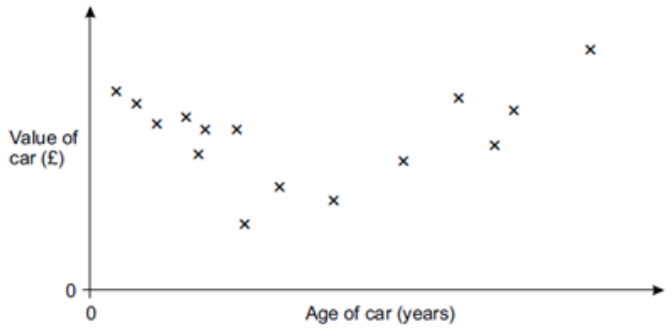
Is Sebastian correct?
Give a reason for your answer.

.....

..... **[2]**

Higher tier – AO2.5

7 This scatter graph shows the values of 15 sports cars plotted against their ages.



(a) (i) Lewis thinks that there is **no correlation** between the ages and values of these cars.

Is Lewis correct?
Give a reason for your answer.

.....

..... [2]

(ii) Sebastian thinks that there is a **relationship** between the ages and values of these cars.

Is Sebastian correct?
Give a reason for your answer.

.....

..... [2]

7	(a)	(i)	The points do not follow the same [linear] pattern	M1		Allow more sophisticated answers such as there is a type of non-linear correlation shown in sections of the graph
			Lewis is correct (no correlation)	A1 1AO2.4a 1AO2.5a		Or allow Lewis is incorrect with the more sophisticated reasoning
		(ii)	The cars decrease in value initially to a certain point but then as the cars get (much) older the graph shows they increase in value	M1		Allow equivalent reasoning but must state both parts of the pattern – decrease in value followed by increase in value
			Sebastian is correct	A1 1AO2.4a 1AO2.5a		

Problem solving in GCSE (9-1) maths

AO3: Solve problems within mathematics and in other contexts			*25% (Foundation Tier)* *30% (Higher Tier)*
Strands	**Elements**	**Coverage**	**Interpretations and definitions**
1 – Translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes	1a – Translate problems in mathematical contexts into a process 1b – Translate problems in mathematical contexts into a series of processes 1c – Translate problems in non-mathematical contexts into a process 1d – Translate problems in non-mathematical contexts into a series of processes	▪ Full coverage over the shortest possible time period (but not in each set of assessments) ▪ Each series should include problems in both mathematical and non-mathematical contexts, as well as both translation into a process and a series of processes – but this may be done in any combination	▪ In the context of this assessment objective, a 'task' is a set of requirements focusing on one problem. ▪ Tasks may be broken down into a number of steps or parts, provided this does not undermine the expectation of students demonstrating their ability to solve problems as a coherent process. ▪ Tasks should place the emphasis on the Learner's own decision-making. They should require Learners to solve problems without the procedures that should be used being clear from the question or task. ▪ The strands of AO3 reflect a problem-solving cycle and as such could be considered as parts of a continuum rather than as independent strands. Each assessment series should provide opportunities for Learners to undertake extended, or multi-step, tasks. ▪ At least one-third of AO3 marks within an assessment series for a single tier should be allocated to tasks which target two or more strands of AO3. Within these multiple-strand tasks, all strands of AO3 should be addressed in each assessment series. ▪ There should be a greater emphasis for both tiers on strands 1/2/3 rather than on strands 4/5. Within strand 1, there should

Problem solving in GCSE (9-1) maths

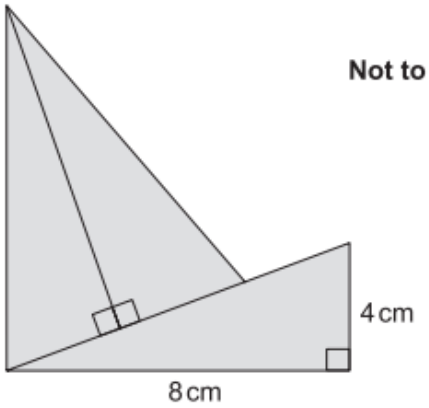
AO3: Solve problems within mathematics and in other contexts			25% (Foundation Tier) 30% (Higher Tier)
Strands	**Elements**	**Coverage**	**Interpretations and definitions**
2 – Make and use connections between different parts of mathematics	*The strand is a single element*	Full coverage in each set of assessments (but not in every assessment)	be a greater emphasis on 1b and 1d than 1a and 1c to ensure an appropriate amount of multi-step problem-solving tasks within each assessment series. ▪ Where relevant, responses should be expected to be presented such that they are within the frame of the original problem rather than in the abstract. ▪ It is possible to have tasks where all the marks are allocated to AO3 but, in such situations, each mark must be awarded against the AO3 strands and elements. It will often be the case that, within a problem-solving task, if the task resolves into a routine procedure and if marks are awarded for the carrying out of that procedure accurately, then those marks must be allocated to AO1. ▪ Responses should not require explanation or justification as this is the focus in AO2, but working should usually be indicated to ensure that partially correct AO3 responses can still be credited. However, it may be appropriate in some cases that partial credit can still be given even where working is not shown – this would be reflected in mark schemes. ▪ Within strand 2, Learners should only be credited for making connections they have generated, rather than any linkages which are explicit in the task. ▪ Within strands 4 and 5, marks may be awarded for methods
3 – Interpret results in the context of the given problem	*The strand is a single element*	Full coverage in each set of assessments (but not in every assessment)	
4 – Evaluate methods used and results obtained	4a – Evaluate methods used	Full coverage in each set of assessments (but not in every assessment)	
	4b – Evaluate results obtained	Full coverage in each set of assessments (but not in every assessment)	

Problem solving in GCSE (9-1) maths

AO3: Solve problems within mathematics and in other contexts			25% (Foundation Tier) 30% (Higher Tier)
Strands	**Elements**	**Coverage**	**Interpretations and definitions**
5 – Evaluate solutions to identify how they may have been affected by assumptions made	*The strand is a single element*	Full coverage in each set of assessments (but not in every assessment)	used, results obtained and/or solutions and assumptions generated by the Learner or provided to the Learner. It should not be understood as pertaining solely to mathematical modelling.

Foundation tier – AO3.1

5 This shape is made from three congruent right-angled triangles.



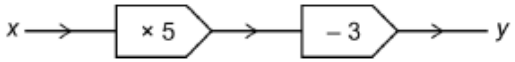
Find the total area of the shape.

..... cm^2 **[3]**

5			48 (cm^2)	**3** **1 AO1.3a** **2 AO3.1b**	**M1** $\frac{1}{2} \times 8 \times 4 = 16$ **M1** *their* '16' × 3	

Higher tier – AO3.1

Here is a function.

Function A: 

(a) (i) Work out y when $x = -2$.

(a)(i) .....[1]

(ii) Work out x when $y = 72$.

(ii) .....[1]

(iii) Find the inverse of function A. **[2]**

Functions (OCR will not include function notation, e.g. f(x), gf(x))

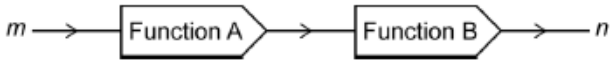
13	**(a)**	**(i)**	-13	**1** 1 AO1.3a		
		(ii)	15	**1** 1 AO1.3a		
		(iii)	$\frac{x+3}{5}$ **oe**	**2** 1 AO1.1 1 AO1.3a	**M1** for correct first step $5x = y + 3$ or a flow diagram with + 3 and ÷ 5	Accept equivalent flow diagram

Higher tier – AO3.1

Here is another function.

Function B: x → × d → + e → y

(b) The diagram below shows a composite function.



When $m = 4$, $n = 53$.
When $m = 9$, $n = 128$.

Find the values of d and e.

(b) d = ...

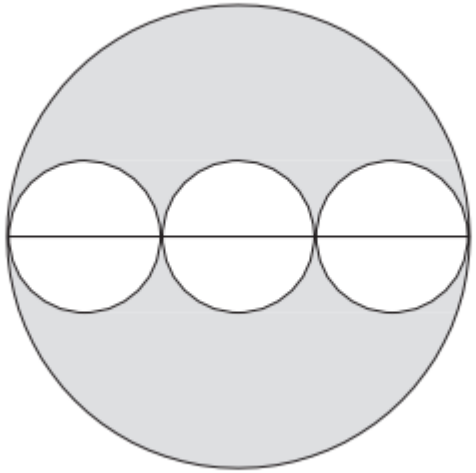
e = ... **[4]**

Functions (OCR will not include function notation, e.g. f(x), gf(x))

Question	Answer	Marks	Part marks and guidance	
(b)	3 2	**4** 1 AO1.3b 3 AO3.1b	**B1** for 17 or 42 **M2** for (128 – 53) ÷ (42 – 17) **oe** or 3 Or **M1** for 128 – 53 or 42 – 17 or 75 or 25	Alternative: **B1** for $17d + e = 53$ **B1** for $42d + e = 128$ **M1** for a subtraction with at most one error e.g. $25d = 75$

Higher tier – AO3.2

Three identical small circles are drawn inside one large circle, as shown in the diagram.
The centres of the small circles lie on the diameter of the large circle.



Find the fraction of the large circle that is shaded.

12			$\frac{2}{3}$	**3** **1 AO1.3a** **1 AO3.1b** **1 AO3.2**	**B1** for radius of large circle = 3 × radius of small circle **M1** for $\frac{9\pi r^2 - 3(\pi r^2)}{9\pi r^2}$ **oe**	

Foundation tier – AO3.3

15 Kieran, Jermaine and Chris play football.

- Kieran has scored 8 more goals than Chris.
- Jermaine has scored 5 more goals than Kieran.
- Altogether they have scored 72 goals.

How many goals did they each score?

15			25, 30, 17	**5** **2 AO1.3a** **2 AO3.1d** **1 AO3.3**	**M1** for any two consistent expressions, e.g. $x - 8$, x **M1** for $x - 8 + x + x + 5 = 72$ **oe** **A1** for $x = 25$ **B1** for Kieran 25 or Jermaine 30 or Chris 17	Accept equivalent correct equations

Foundation tier – AO3.4/AO3.5

5 Darren says

I can run 100 m in 15 seconds, so I should be able to run 800 m in 120 seconds.

Do you think that he would take more or less than 120 seconds to run 800 m?
Explain your answer, with reference to any assumptions Darren has made.

.....

.....

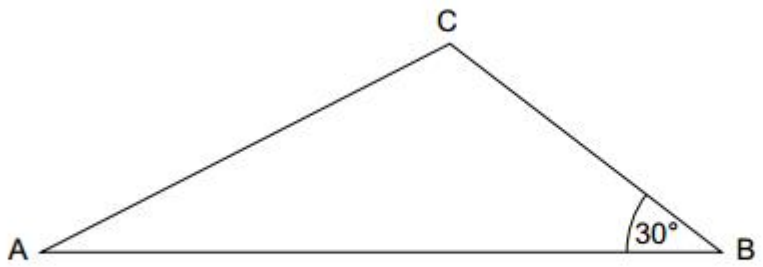
.....

..... **[3]**

5			He has assumed he can run 800 m at the same speed as he can run 100 m, but he will run 800 m at a slower speed, therefore it will take him more than 120 s	**3** **1 AO2.1a** **1 AO3.4a** **1 AO3.5**	**B1** for correct reference to Darren's assumption OR $\frac{100}{15} = \frac{800}{120}$ **soi** **B1** for 'his speed will be slower over 800 m' **oe**	

Higher tier – Multiple AOs

Triangle ABC has area 40 cm^2.
AB = 2BC.



Not to scale

Work out the length of BC.
Give your answer as a surd in its simplest form.

Question	Answer	Marks	Part marks and guidance	
16	$4\sqrt{5}$	6 2AO1.3b 1AO3.1b 2AO3.2 1AO3.3	**B5** for $\sqrt{80}$ **oe** OR **M4** for $\sqrt{\frac{40}{0.5\times 2\times \sin 30}}$ **oe** Or **M3** for $2x^2 = \frac{40}{0.5\sin 30}$ **oe** Or **M2** for $\frac{1}{2}x \times 2x \sin 30 = 40$ **oe** And **B1** for sin 30 = 0.5	Allow use of any variable **B1** may be awarded with **M4**, **M3** or **M2**

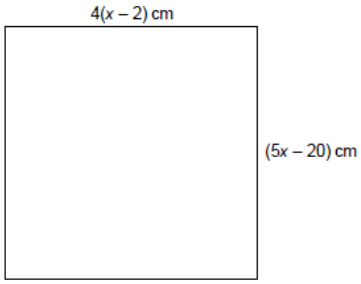
GCSE (9-1) maths Assessment Objectives

- Take a look at the GCSE (9-1) sample questions.
- Which Assessment Objectives are they targeting...?

1 Bethany says that $(2x)^2$ is always greater than or equal to $2x$.

Decide whether she is correct or not.
Show your working to justify your decision.

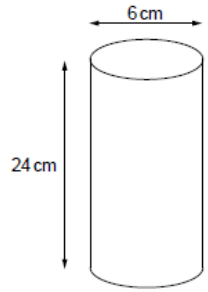
2 This is a square.



Not to scale

Work out the length of the side of the square.

3 Four solid balls are packed in a cylindrical container.



The diameter of each ball is 6 cm.
The cylinder has diameter 6 cm and height 24 cm.

Calculate the volume of unused space in the cylinder.

[The volume V of a sphere is $V = \frac{4}{3}\pi r^3$ where r is the radius.]

1 Bethany says that $(2x)^2$ is always greater than or equal to $2x$.

Decide whether she is correct or not.
Show your working to justify your decision. **[3]**

Question	Answer	Marks	Part marks and guidance
	e.g. When $x = 0.1$ $(2x)^2 = 0.04$ $2x = 0.2$ So $(2x)^2 < 2x$ which contradicts Bethany's statement So it is not always true	3 2 AO2.4a 1 AO2.5a	**M1** for attempting to demonstrate that for some value of x in range $0 < x < \frac{1}{2}$ it is not true **A1** for complete working **A1** for explanation or **M1** for attempt including squaring bracket **A1** for complete solution for either $x < 0$ or $x \geq \frac{1}{2}$ **A1** for explanation or **B1** for a counter example given without working

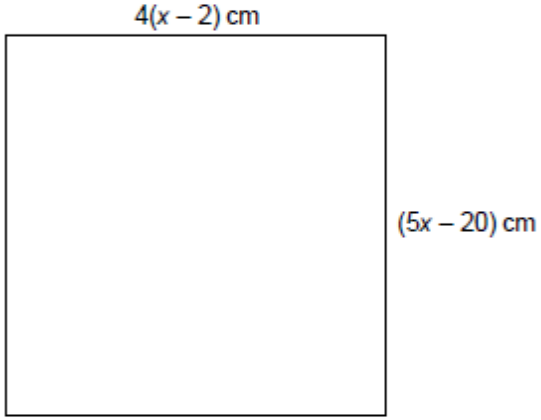
AO2.4 – Present arguments and proofs

- 2.4a – Present arguments

AO2.5 – Assess the validity of an argument & critically evaluate a given way of presenting information

- 2.5a – Assess the validity of an argument

2 This is a square.



Not to scale

Work out the length of the side of the square.

Question			Answer	Marks	Part marks and guidance	
3			40	5 1AO1.3b 3AO3.1b 1AO3.3	**M1** for $4(x – 2) = 5x – 20$ **M1** for $4x – 8 = 5x – 20$ AND **M2** for $x = 12$ Or **M1** for one correct step solving equation	

AO1.3 – Accurately carry out routine procedures or set tasks requiring multi-step solutions

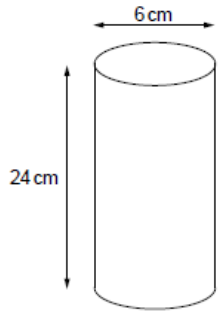
- 1.3b – Accurately carry out set tasks requiring multi-step solutions

AO3.1 – Translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes

- 3.1b – Translate problems in mathematical contexts into a series of processes

AO3.3 – Interpret results in the context of the given problem

3 Four solid balls are packed in a cylindrical container.



The diameter of each ball is 6 cm.
The cylinder has diameter 6 cm and height 24 cm.

Calculate the volume of unused space in the cylinder.

[The volume V of a sphere is $V = \frac{4}{3}\pi r^3$ where r is the radius.]

Question	Answer	Marks	Part marks and guidance	
4	226[.2] or 72π	1AO1.3b 1AO2.3a 4AO3.1d	**B3** for 678.58 or 216π OR **M1** for 9π **M1** for *their* '9π' × 24 **soi** AND **B1** for 113.1 or 113.097 or 36π **M1** for *their* '113.1' × 4	

AO1.3 – Accurately carry out routine procedures or set tasks requiring multi-step solutions
- 1.3b – Accurately carry out set tasks requiring multi-step solutions

AO2.3 – Interpret and communicate information accurately
- 2.3a – Interpret information accurately

AO3.1 – Translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
- 3.1d – Translate problems in nonmathematical contexts into a series of processes

Introducing reasoning skills in maths lessons

- Present a chart/diagram and ask students to discuss what it can tell us, or give criticisms.
- Present statements and ask students to say if each one is always/sometimes/never true.
- Use sample GCSE (9-1) questions as starters.

Introducing reasoning and problem solving skills in maths lessons

- Start with standard concepts and techniques, but move on to applying them in reasoning and problem solving questions as quickly as possible.

- Pick the right question for the students.

- Give support at the right time and give hints rather than clear solutions - maximise student's own sense of achievement in reaching an answer.

Introducing reasoning and problem solving skills in maths lessons

- Group work, or questioning.

- Positive mistakes.
  - Use them as an opportunity for students to explore differing responses.

- Consider assumptions.
  - Change numbers and look at the effect.

Problem Solving in maths lessons

- Starter activities/puzzles

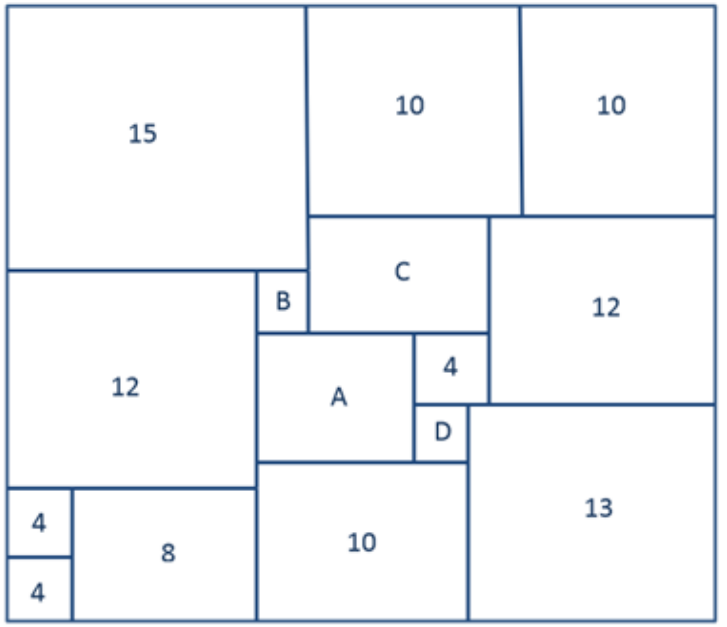
FMSP GCSE Problem Solving Resources

Problem 4

Each shape contained within the largest square is also a square.

The number in each square gives the length of its sides.

What are the values of A, B, C and D?



http://www.furthermaths.org.uk/manager_area/files/Problem_4.pdf (OCR6.03a)

Problem Solving in maths lessons

- Plenary Activities

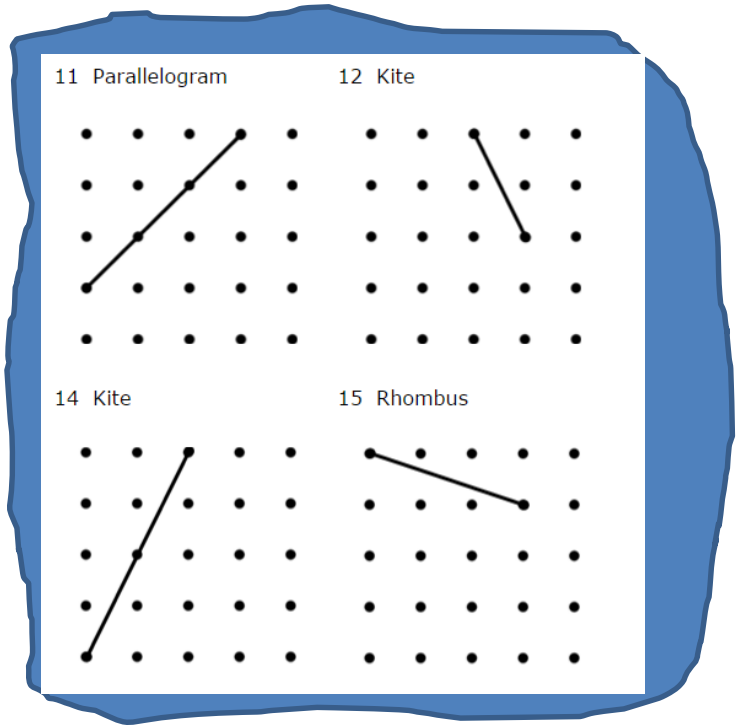
Complete the Quadrilateral

Stage: 3 ★

Join dots on each grid below to make the named quadrilateral.
You must use the side given, you can't shorten or extend it.

If there is more than one possibility, try to find the one with the largest area.

Try not to resort to special cases. For example, a parallellogram should not be a rhombus, a rectangle or a square.



http://nrich.maths.org/11234 (OCR8.04b)

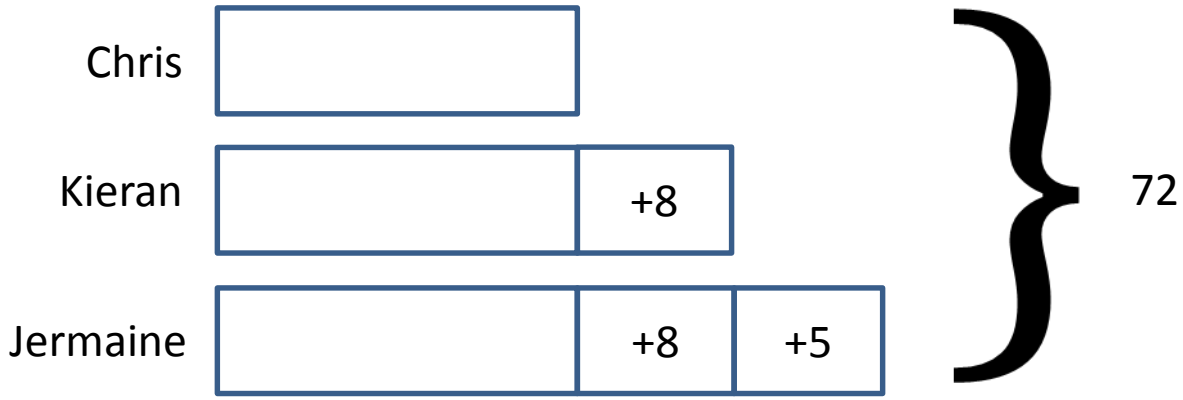
Problem Solving in maths lessons

- Introduce new concepts or skills

Kieran, Jermaine and Chris play football.

- Kieran has scored 8 more goals than Chris.
- Jermaine has scored 5 more goals than Kieran.
- Altogether they have scored 72 goals.

How many goals did they each score?



http://www.thesingaporemaths.com/stratf.html

Problem Solving in maths lessons

- Explore maths beyond the question.

How many different ways can you make a rectangle with an area of 12 cm^2 if the sides are integers?

-Investigate rectangles with different areas?

-Which areas make the fewest rectangles?

-Which areas have an odd number of rectangles?

-…?

Problem Solving in maths lessons

- Applications of maths in context

Stadium Evacuation!

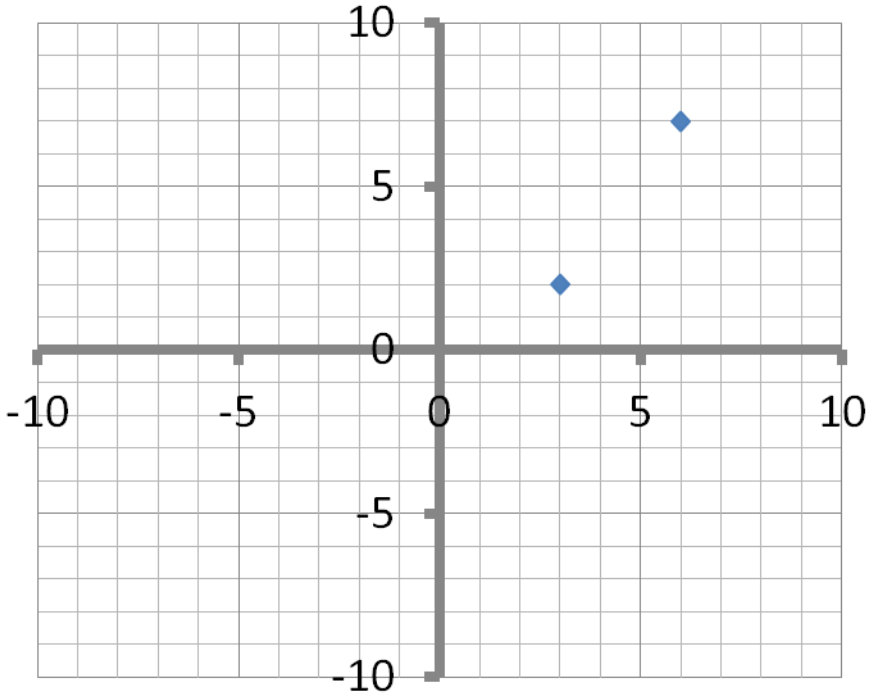
http://www.bbc.co.uk/education/clips/zxcc7ty

Calculate how many double doors are required.

- Look at rate of movement, changing units and discuss speed of evacuation.
- Investigate what would happen to the speed of the evacuation if fewer doors were available.

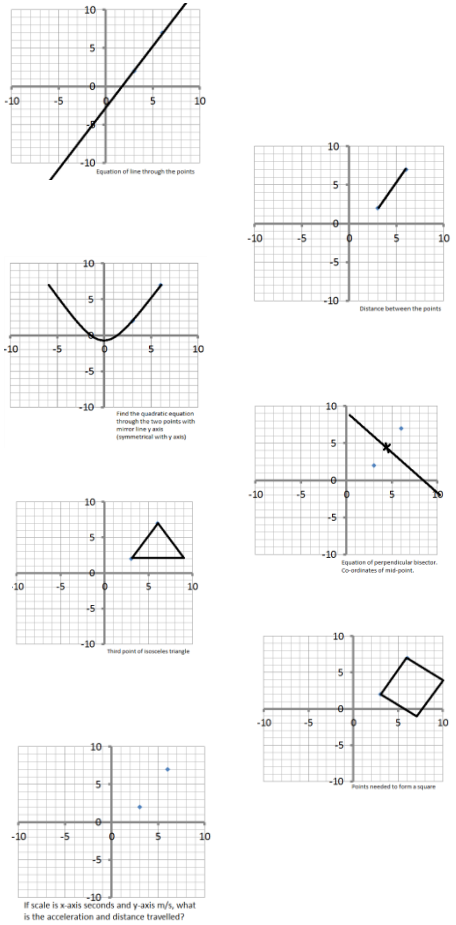
Problem Solving in maths lessons

- What's the question?



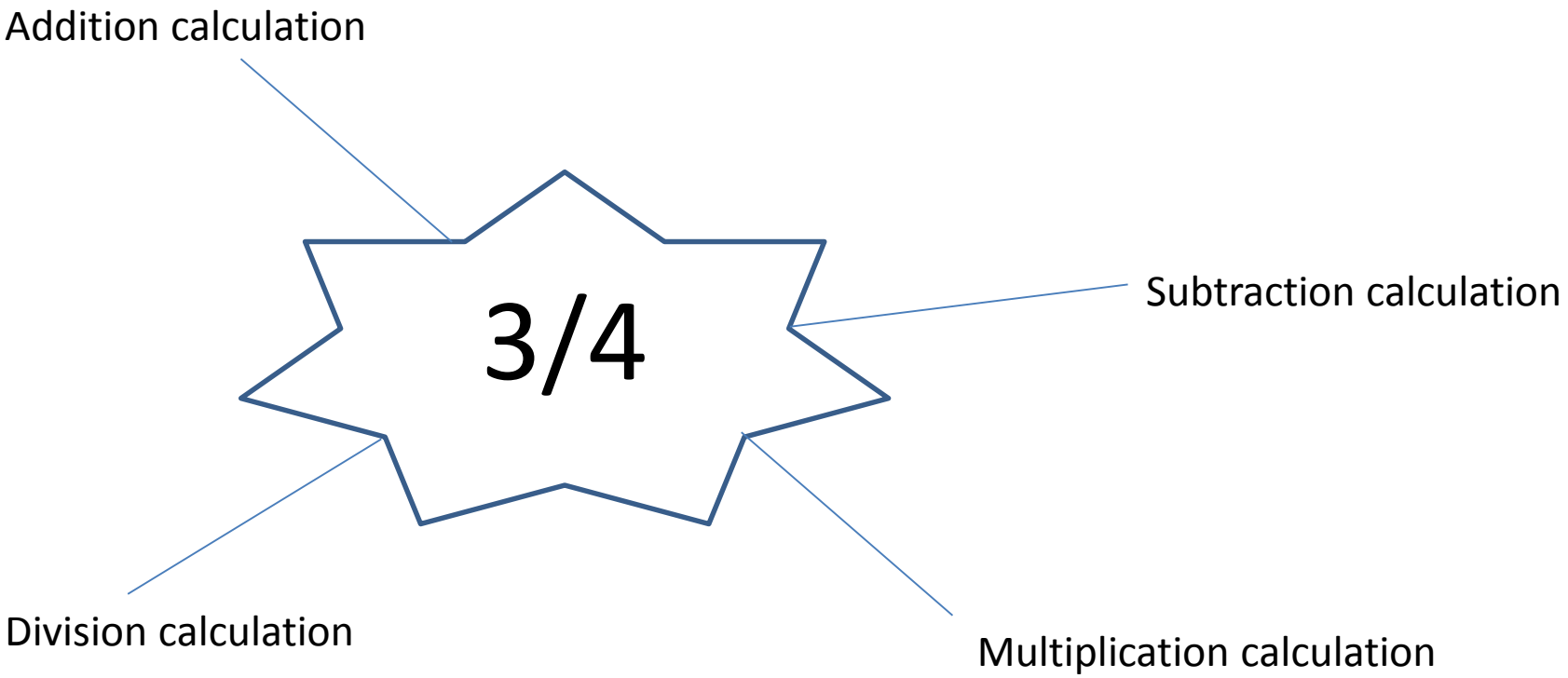
Problem Solving in maths lessons

- What's the question?
  - Find the equation of the line through the two points.
  - Find the distance between the two points.
  - Find a quadratic equation through the points.
  - Find the equation of a line perpendicular to the line through the two points.
  - Find a third point to form an isosceles triangle.
  - Find the coordinates that would make it into a square.
  - If the *x*-axis is time and the *y*-axis speed, find the acceleration / distance travelled.



Problem Solving in maths lessons

- Revision activities



Reasoning and problem solving in maths lessons

- Adapting GCSE questions to meet needs.
  - Lessen demand?
  - Increase demand?

Ema has done some calculations.

For each calculation, explain how you know the answer is wrong without working out the correct answer.

(a) $0.38 \times 0.26 = 0.827$

.....

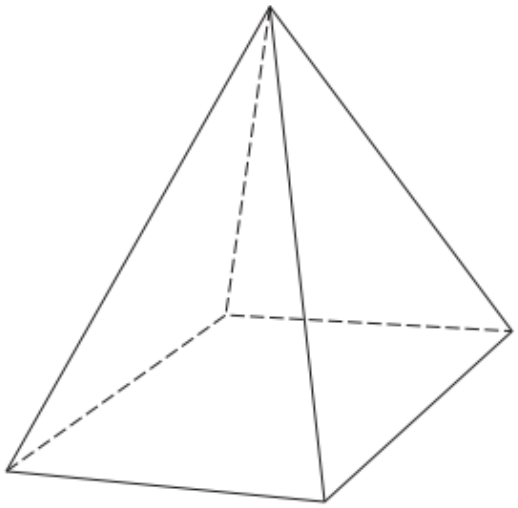
..... **[1]**

(b) $\frac{3}{4}+\frac{2}{3}=\frac{5}{7}$

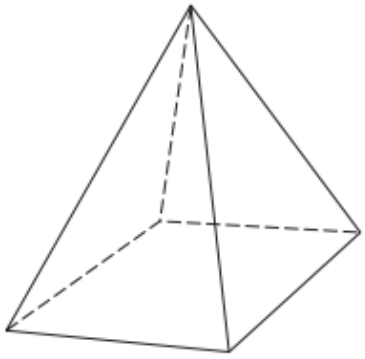
.....

.....

Two similar pyramids A and B have surface areas $180\,cm^2$ and $80\,cm^2$ respectively.



Pyramid A



Pyramid B

The volume of pyramid A is $810\,cm^3$.

Show that the volume of pyramid B is $240\,cm^3$. **[5]**

Reasoning and problem solving resources

- www.ocr.org.uk/gcsemaths
  - Sample Assessment Materials and Practice Papers list the AOs allocated to each question in the mark scheme.
  - Curriculum Planners contain AO2 and AO3 suggestions for each topic.
  - Curriculum Planners, Delivery Guides and Transition Guides contain links to resources, including conceptual and contextual.

Reasoning and problem solving resources

- www.ocr.org.uk/gcsemaths
  - Check In tests contain AO2 and AO3 questions on each topic.

6. Zosia says "6 + 5 × 2 is equal to 22."
   Explain why Zosia is incorrect.

7. Explain why $(4 - 2) \div (6 - 3)$ could be written as $\frac{2}{3}$.

8. If the reciprocal of 5 is $\frac{1}{5}$ and the reciprocal of $\frac{1}{3}$ is 3, explain how you could find the reciprocal of $\frac{1}{2}$.

9. John makes party bags containing 1 ball, 2 sweets and 1 card. If each ball costs 50p, each sweet costs 5p and each card costs 15p, how much change will he have from £10 if he makes up 8 bags?

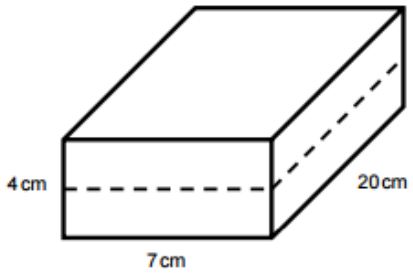
10. Arrange the following in order from smallest to largest.

$$\frac{4+2}{1+3} \qquad \frac{(3+1)^2}{4} \qquad \frac{3+1}{4\times 2} \qquad \frac{(3-4)^2}{1}$$

6. A small cube has sides of length 2 cm and surface area 24 cm^2. 8 small cubes are put together to make a larger cube with 4 cm sides. How many times larger is the surface area of the new cube? Explain your answer.

7. William has a cylinder of height 14 cm and diameter 6 cm. He calculates the curved surface area as $2 \times \pi \times 6 \times 14$. Explain what he has done wrong.

8. A cuboid has length 20 cm, width 7 cm and height 4 cm. It is cut in half horizontally as shown on the diagram below. What is the difference between the surface area of the original shape, and the total surface area of the two pieces?



9. Sam has a watering can holding 5 litres of water. He needs to fill some cylindrical jars with a base of 81 cm^2 to a depth of 6 cm. How many pots can he completely fill?

10. A cylinder has a cross-section with radius 3 cm and surface area 435 cm^2. Calculate the length of the cylinder.

Reasoning and problem solving resources

- http://nrich.maths.org/public/leg.php?group_id=8

- http://www.nuffieldfoundation.org/key-ideas-teaching-mathematics

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