The Boolean Maths Hub Conference

January 14th, 2017

Workshop 1: Teaching for Mastery at Secondary

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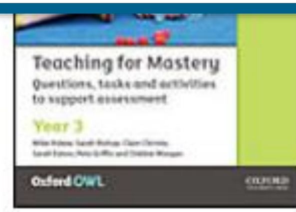
What do we mean by mastery?

The essential idea behind mastery is that *all children*[2] need a ***deep*** understanding of the mathematics they are learning

Year



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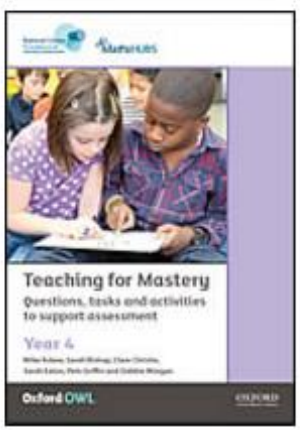
Year Four



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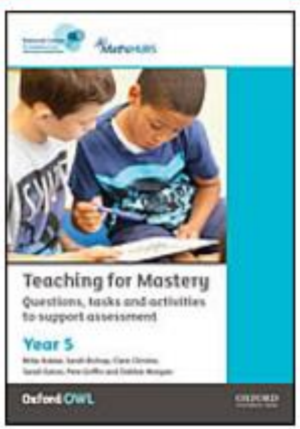
Year Five



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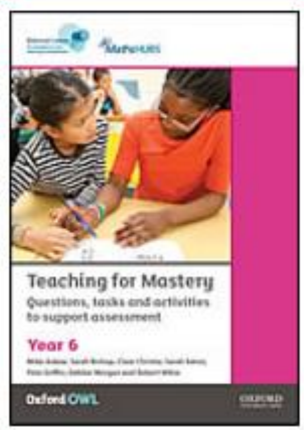
Year Six



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Teaching for Mastery Assessment Materials:

https://www.ncetm.org.uk/resources/46689

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What is 'Mastery'?

1. A mastery approach; a set of principles and beliefs.
2. A mastery curriculum.
3. Teaching for mastery: a set of pedagogic practices.
4. Achieving mastery of particular topics and areas of mathematics.

What is Mastery?

Mastery means that learning is sufficiently:

- Embedded
- Deep
- Connected
- Fluent

In order for it to be:

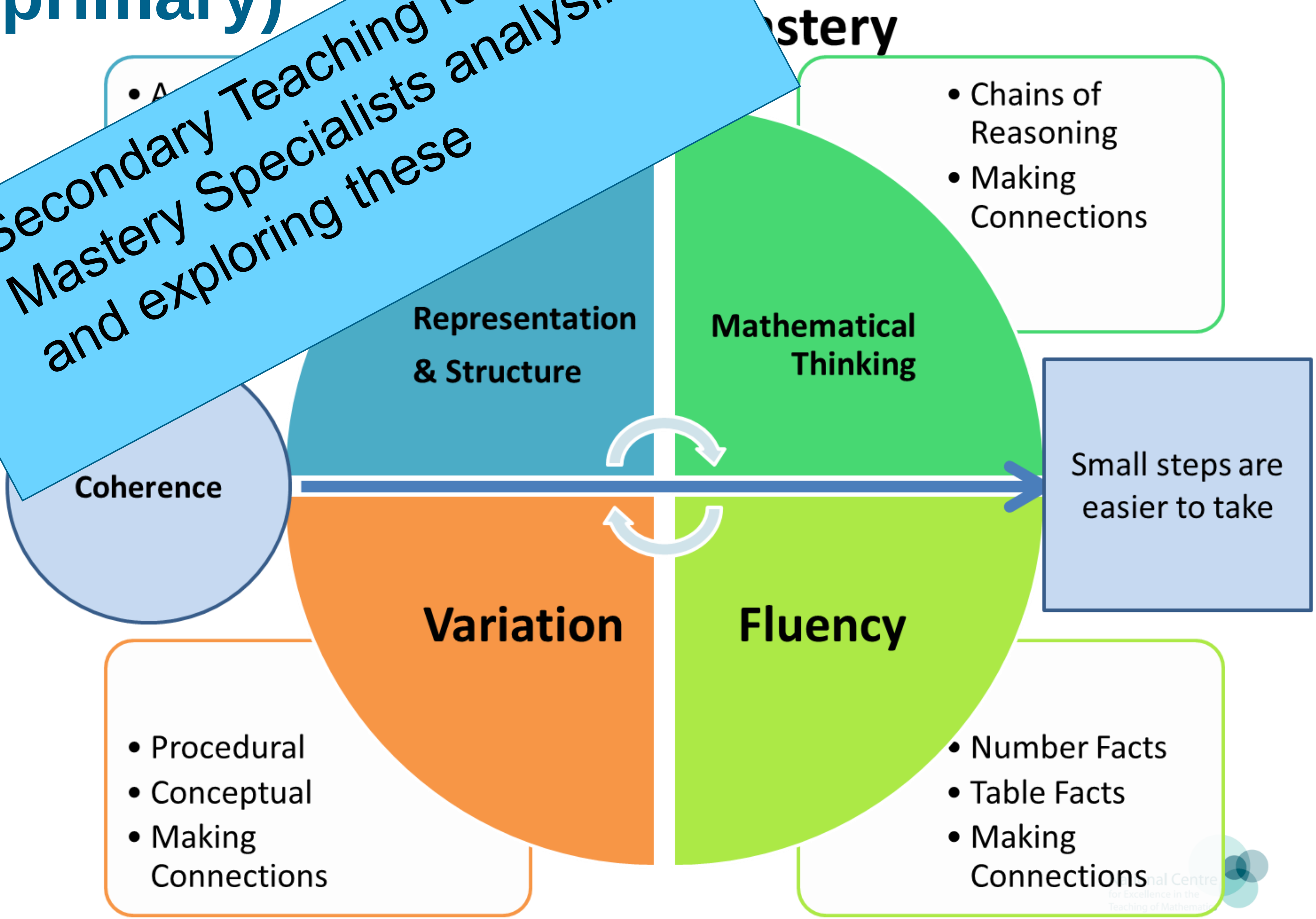
- Sustained
- Built upon
- Connected to

Essence of Mastery

- Lessons involve tasks interspersed with questioning, explanation, demonstration, reasoning and discussion.
- Procedural fluency and conceptual understanding are developed in tandem through **intelligent practice.**
- Significant time is spent developing deep knowledge of the key ideas needed to underpin future learning.
- Structures and connections are emphasised.
- Key facts are learnt to avoid cognitive overload and to enable pupils to focus on new concepts.

Five big ideas (at primary)

Secondary Teaching for Mastery Specialists analysing and exploring these



The Secondary Teaching for Mastery Specialist Programme

Balancing conceptual understanding and procedural fluency: Coherence

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Coherence

- In mathematics, new ideas, skills and concepts build on earlier ones.
- If you want build higher, you need strong foundations.
- Every stage of learning has key conceptual pre-cursors which need to be understood deeply in order to progress successfully.
- When something has been deeply understood and mastered, it can and should be used in the next steps of learning.

Conceptual vs procedural knowledge

Key Question:
"What do students need to understand in order to do something fluently?"

Mathematical knowledg … s
significant, fundamental … l
and procedural knowledge. St…
competent in mathematics if either k…
deficient or if they both have been acquire…
separate entities.

When concepts and procedures are not connected, students may have a good intuitive feel for mathematics but not solve the problems, or they may generate answers but not understand what they are doing.

Heibert and LeFevre - Conceptual and procedural knowledge in mathematics: an introductory analysis (Chap 1 of 'Conceptual and procedural knowledge: the case of mathematics' (1986).

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There is nothing to fear about the ability to execute a correct mathematical procedure with ease, i.e., without thinking.

… what one must fear is limiting one's mastery of such procedures to only the mechanical aspect and ignoring the mathematical understanding of why the procedures are correct.

H Wu: American Educator 2011

2016 Key Stage 2 SATs

3 $326 \div 1 =$

18 $122{,}456 - 11{,}999 =$

33 $\frac{3}{5} \div 3 =$

3 Write the three missing digits to make this **addition** correct.

21 $5{,}542 \div 17 = 326$

Explain how you can use this fact to find the answer to **18 × 326**

What are the issues with algorithms?

- What is the essential knowledge pupils need in order to understand particular algorithms?
- Have pupils fully mastered the underlying mathematical structures ***before*** they are introduced to an algorithm?
- Do we try to give children both the structures and the procedures at the same time?

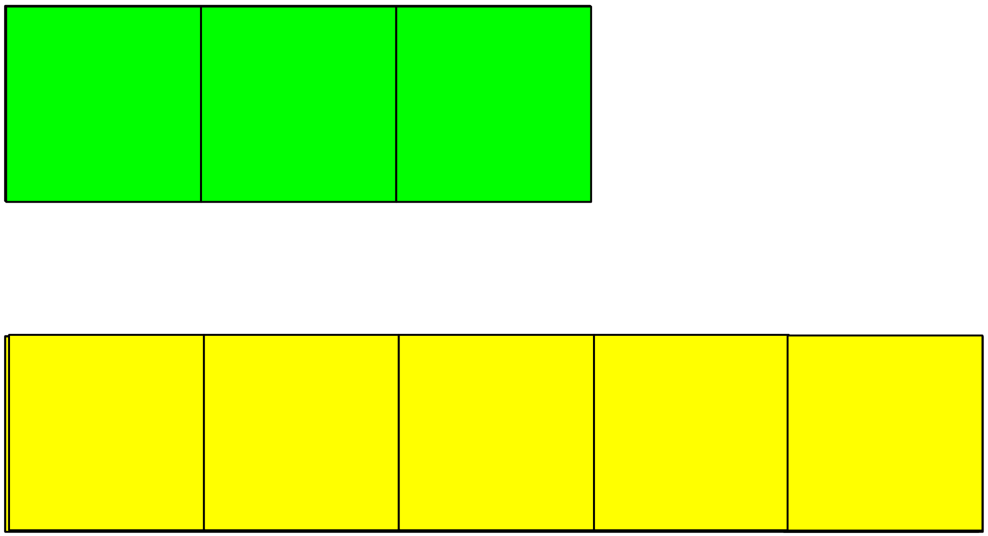
Division of fractions

"Turn it upside down and multiply"!!

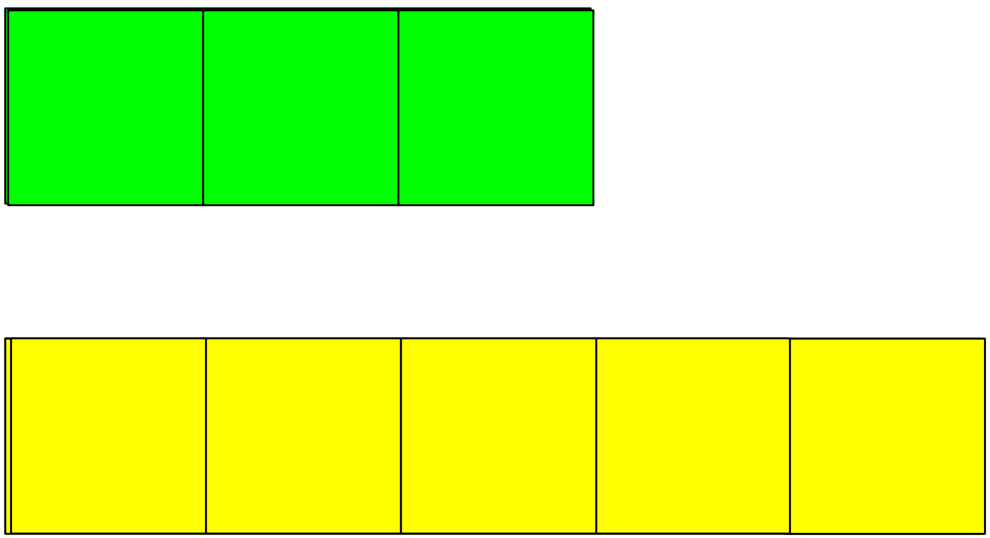
Why?

Division

(as multiplicative comparison)



$$\frac{3}{5} = 3 \div 5$$

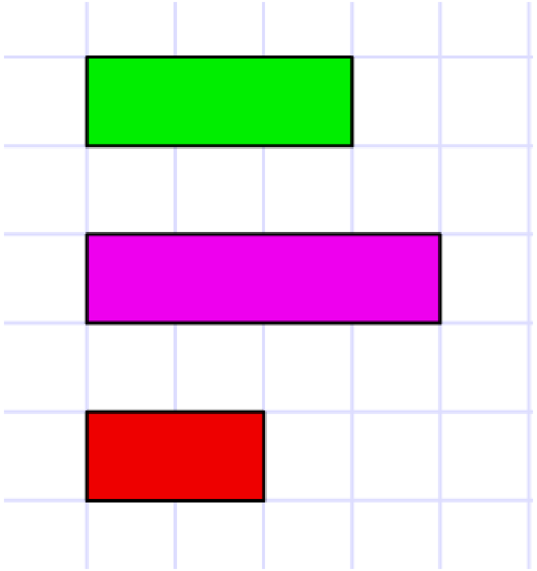


$$\frac{5}{3} = 5 \div 3$$

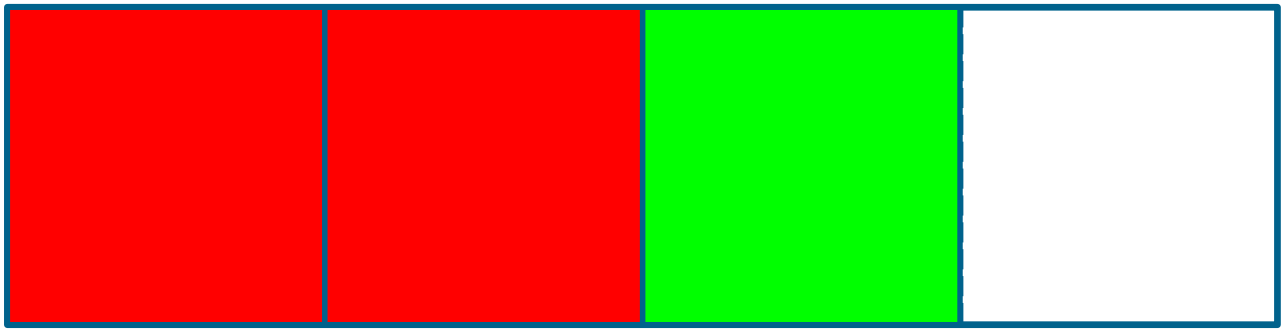
Division of fractions

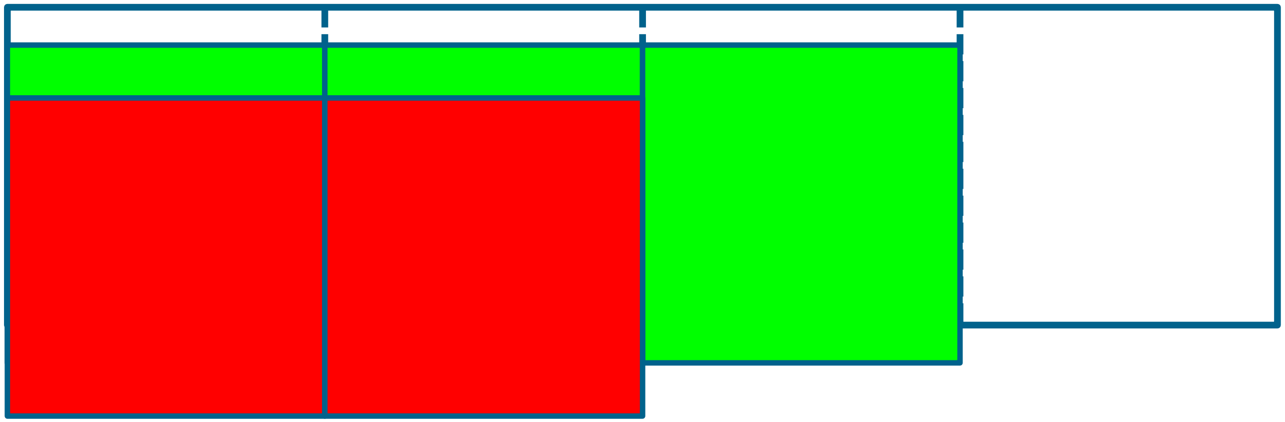
(multiplicative comparison of fractions)

$$\frac{3}{4} \div \frac{1}{2}$$



How does this diagram support seeing the meaning of this division?





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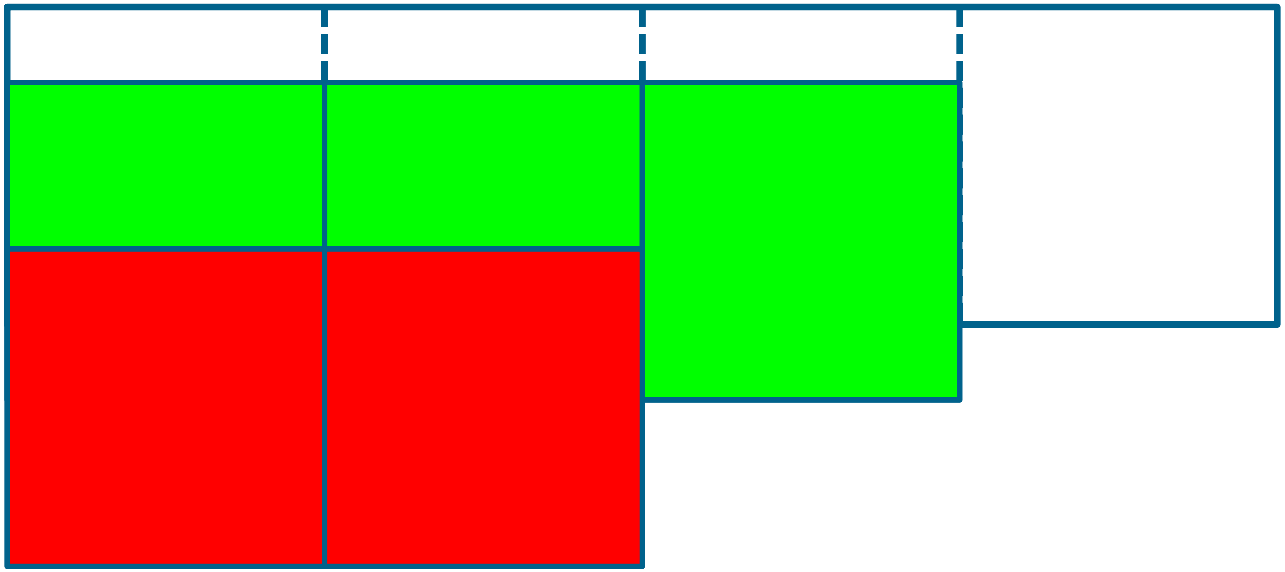

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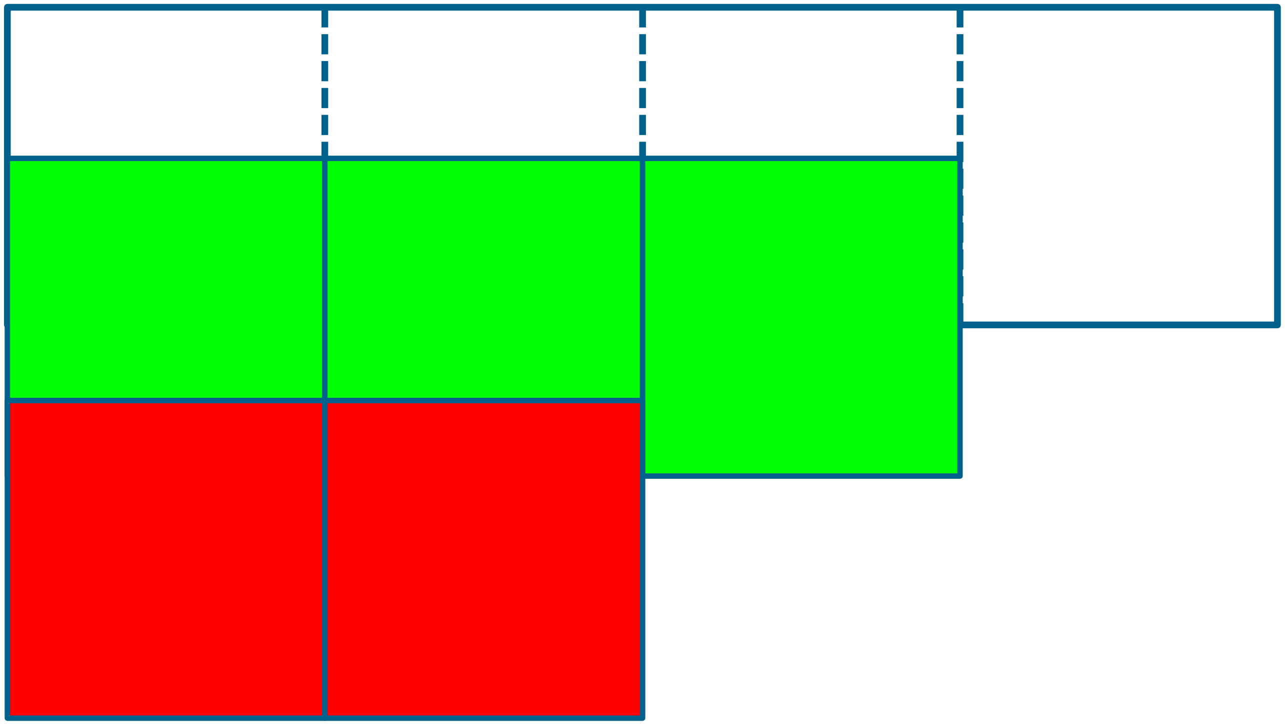

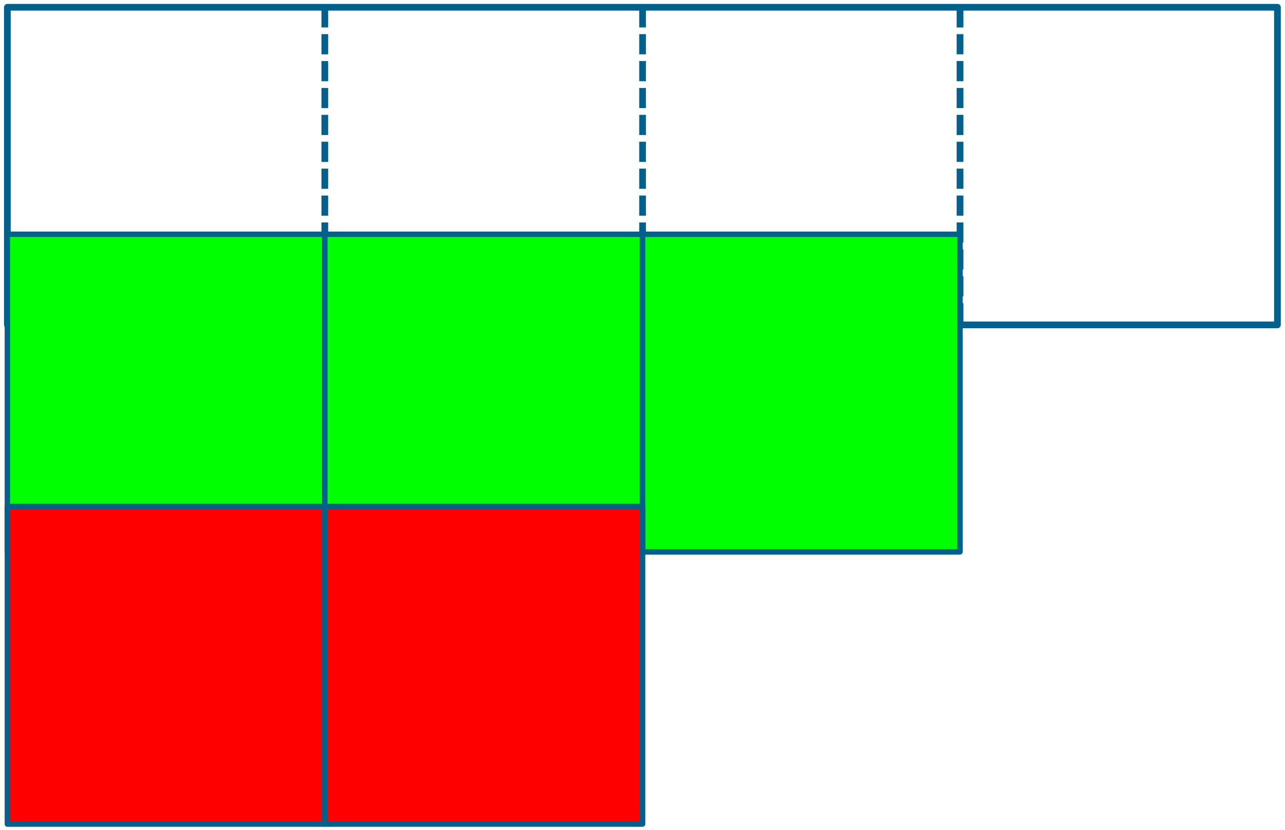

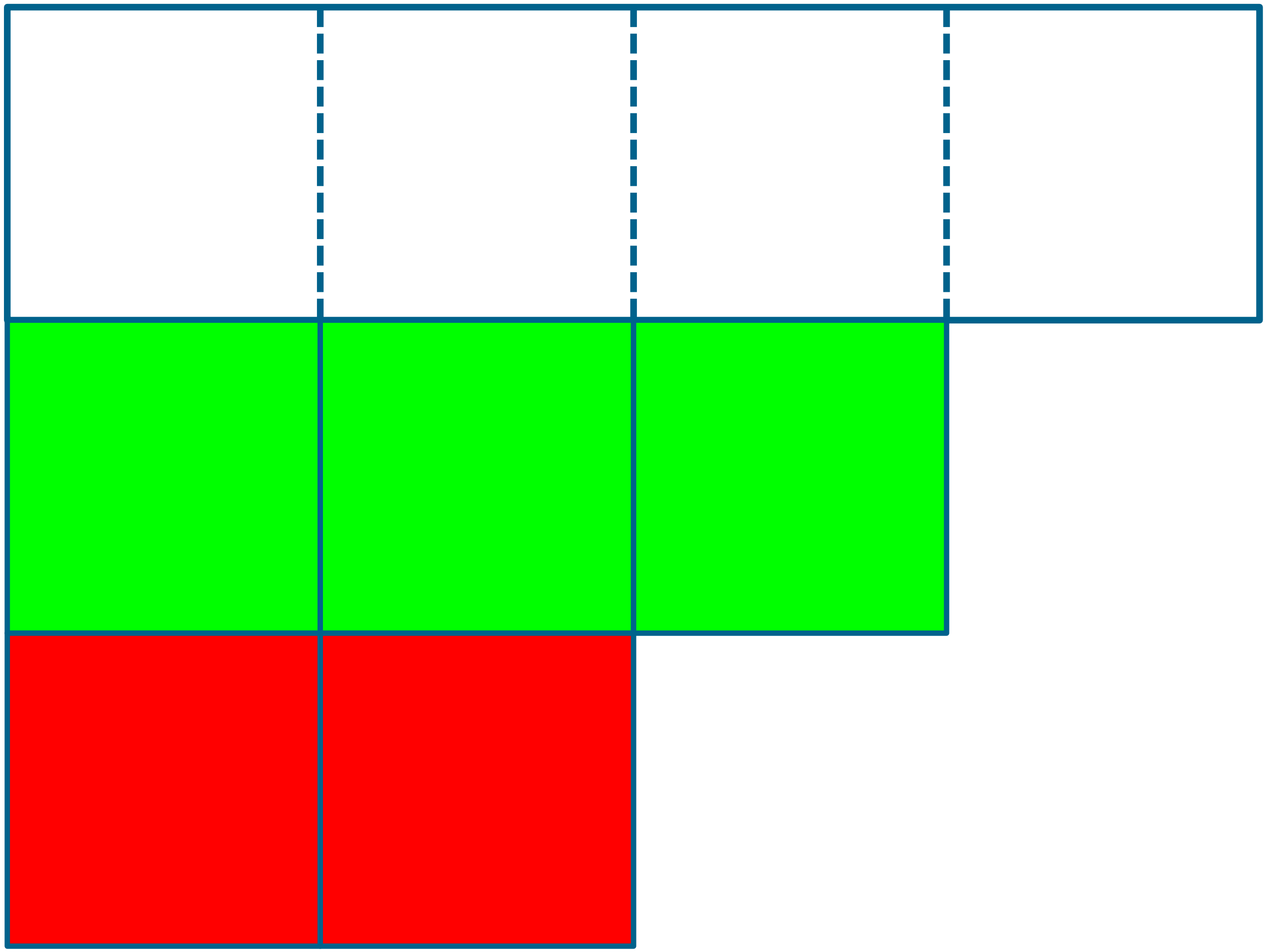

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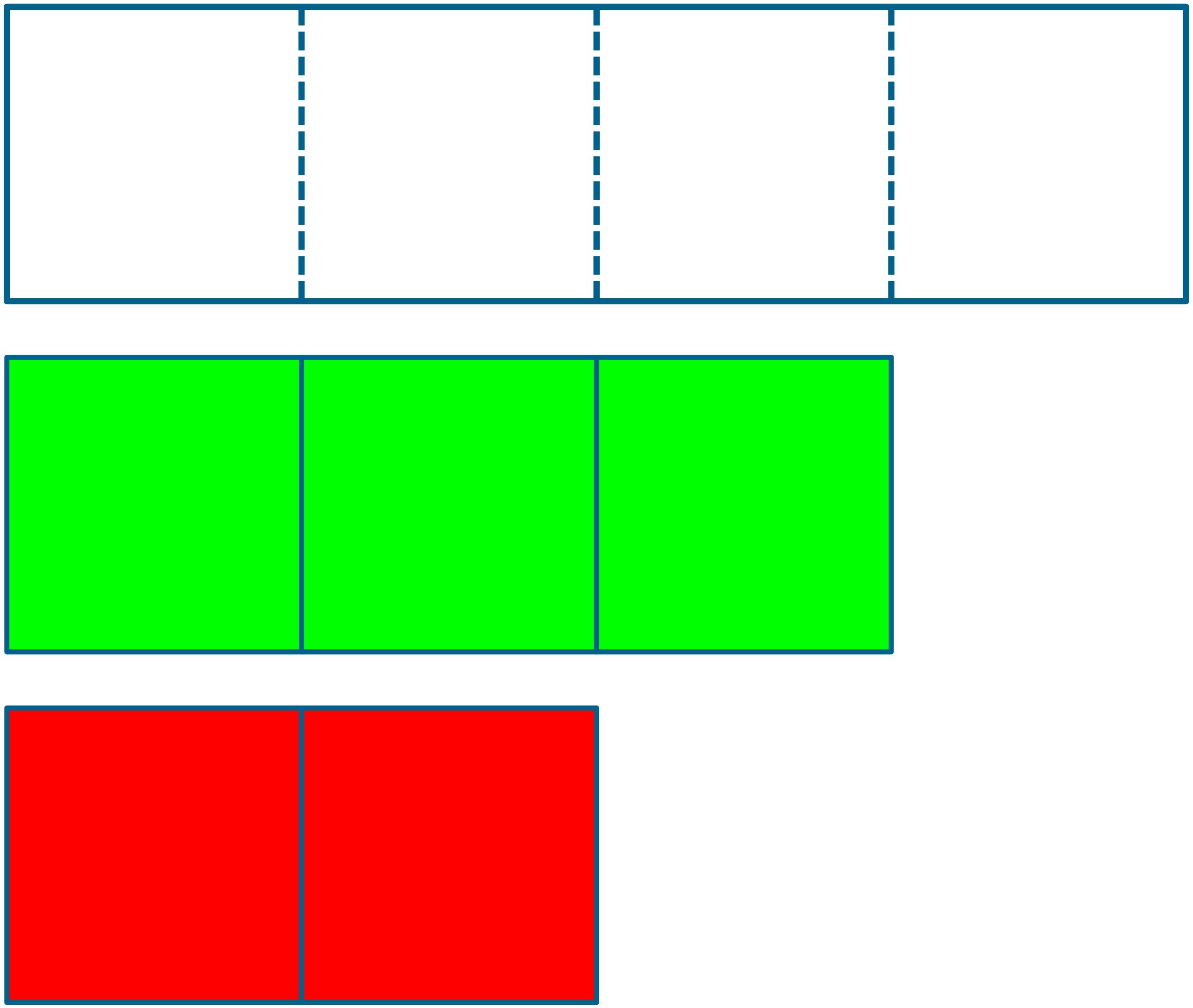

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Understanding procedures ← Teaching structures

- Have pupils fully mastered the underlying mathematical structures ***before*** they are introduced to an algorithm/procedure?
- Do we try to give children both the structures and the procedures at the same time?
- What is the essential knowledge pupils need in order to understand particular algorithms/procedures?
- Teaching structures leads to understanding including understanding of (and, therefore fluency with) procedures.

The Secondary Teaching for Mastery Specialist Programme

Variation … an introduction

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$$9999 + 999 + 99 + 9 + 5 =$$

$$0.62 \times 37.5 + 3.75 \times 3.8 =$$

$$\left(\frac{4}{5} + \frac{1}{6}\right) + \left(\frac{5}{6} + \frac{1}{7}\right) + \left(\frac{6}{7} + \frac{1}{8}\right) + \left(\frac{7}{8} + \frac{1}{9}\right) + \left(\frac{8}{9} + \frac{2}{10}\right) =$$

What are you attending to?

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Simultaneous Equations

New Questions | Show Answers

Bronze

Solve:

Q1) $5x - 5y = 5$
$5x - 6y = -3$

Q2) $8x - 8y = 8$
$8x - 4y = 40$

Q3) $4x + 10y = 46$
$4x - 8y = -8$

Q4) $6x - 4y = 22$
$6x - 8y = 2$

Q5) $3x - 5y = 11$
$3x + 8y = 37$

Q6) $3x - 6y = 3$
$3x + 5y = 47$

Q7) $7x - 8y = 6$
$7x - 5y = 30$

Q8) $3x + 6y = 42$
$3x - 8y = -14$

Silver

Solve:

Q1) $2x + 9y = 48$
$3x - 7y = -10$

Q2) $3x - 3y = 6$
$2x + 9y = 48$

Q3) $2x + 5y = 33$
$4x + 3y = 45$

Q4) $2x + 3y = 24$
$3x + 6y = 42$

Q5) $7x - 6y = -19$
$-7x + 2y = 39$

Q6) $-8x - 2y = -44$
$-3x - 7y = 21$

Q7) $-9x + 8y = 40$
$-5x + 5y = 20$

Q8) $10x + 8y = 18$
$7x + 7y = 7$

Gold

Solve:

Q1) $y = 8x + 14$
$y = 7x + 11$

Q2) $y = 4x + 4$
$y = 8x + 16$

Q3) $y = -3x - 12$
$5y + 5x = 10$

Q4) $y = -2x + 16$
$y = 2x - 8$

Q5) Find the coordinates of the point of intersection of:
$y = 2x - 3$
$y = 5x + 6$

Q6) Find the coordinates of the point of intersection of:
$y = -7x + 17$
$y = 2x - 10$

What are these examples examples of?

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Variation versus variety

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Variety

- ‘Pick and mix’
- Most practice exercises contain variety

Variation

- Careful choice of WHAT to vary
- Careful choice of what the variation will draw attention to

2011, Mike Askew, Transforming Primary Mathematics

Variation Theory in Practice

What are you attending to?

What are these examples of?

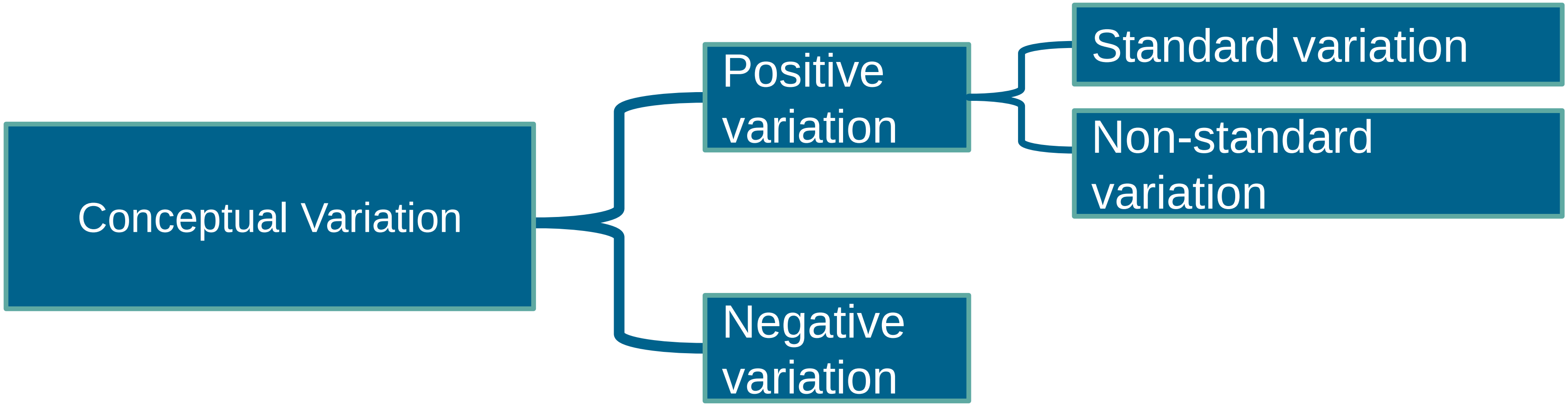
Set A	Set B
120 – 90	120 – 90
235 – 180	122 – 92
502 – 397	119 – 89
122 – 92	235 – 180
119 – 89	237 – 182
237 – 182	502 – 397

Taken (and slightly modified) from Mike Askew, Transforming Primary Mathematics, Chapter 6

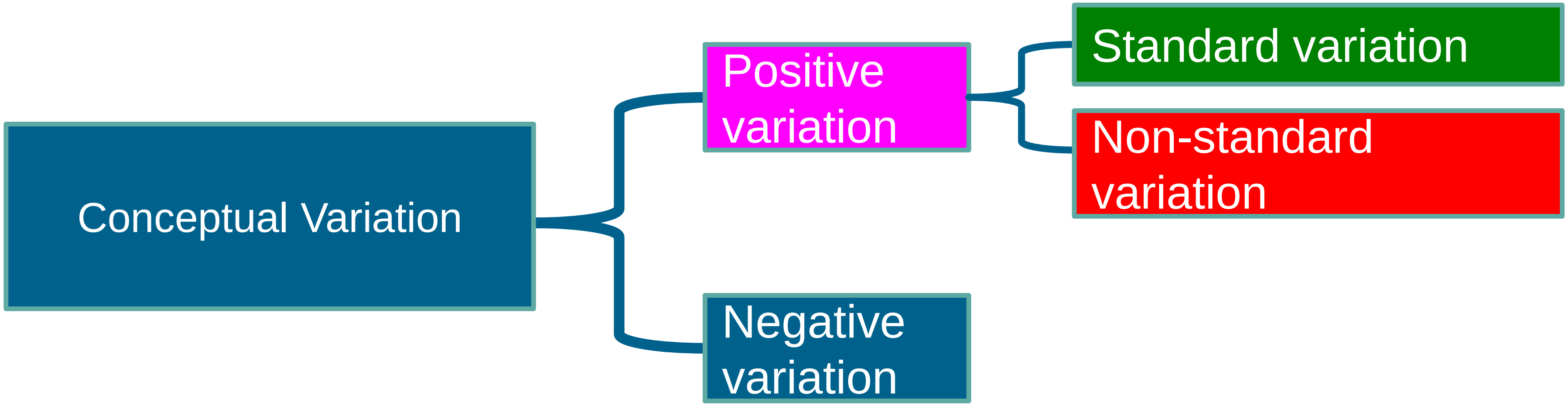
Conceptual and procedural variation

- Conceptual Variation
  - Positive variation
    - Standard variation
    - Non-standard variation
  - Negative variation
- Procedural Variation
  - Variation to scaffold mathematical thinking
  - Variation as a support for problem solving

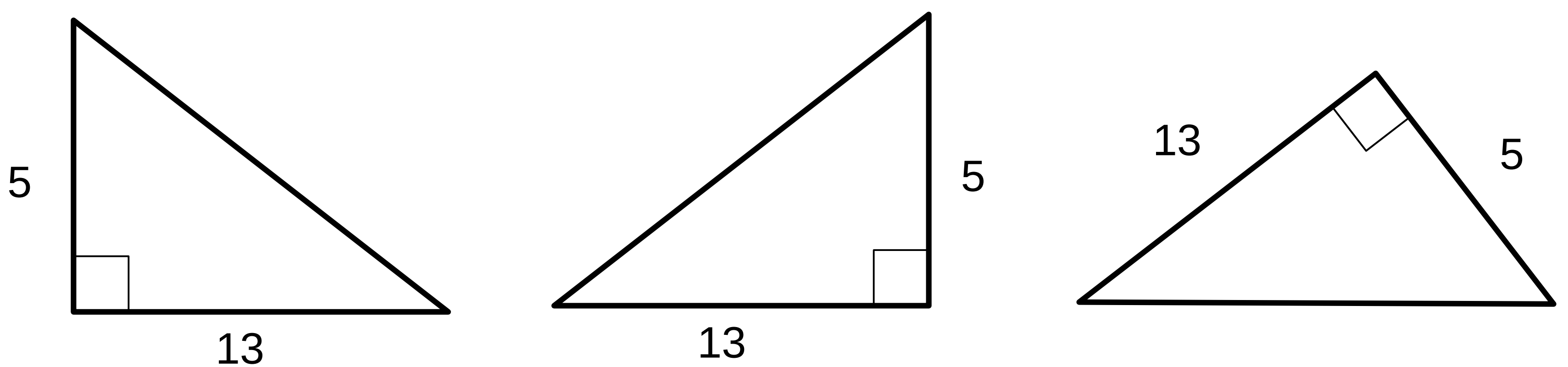
Conceptual and procedural variation



Conceptual and procedural variation



Find the length of the missing side:



Standard and Non-standard

Q1 Solve the following simultaneous equations:

a) $4x + 6y = 16$

$x + 2y = 5$

b) $3y - 8x = 24$

$3y + 2x = 9$

c) $3y - 10x - 17 = 0$

$\frac{1}{3}y + 2x - 5 = 0$

d) $\frac{x}{2} - 2y = 5$

$12y + x - 2 = 0$

e) $3x - 4y = 5x - 14$

$2y + x = 11y - 26$

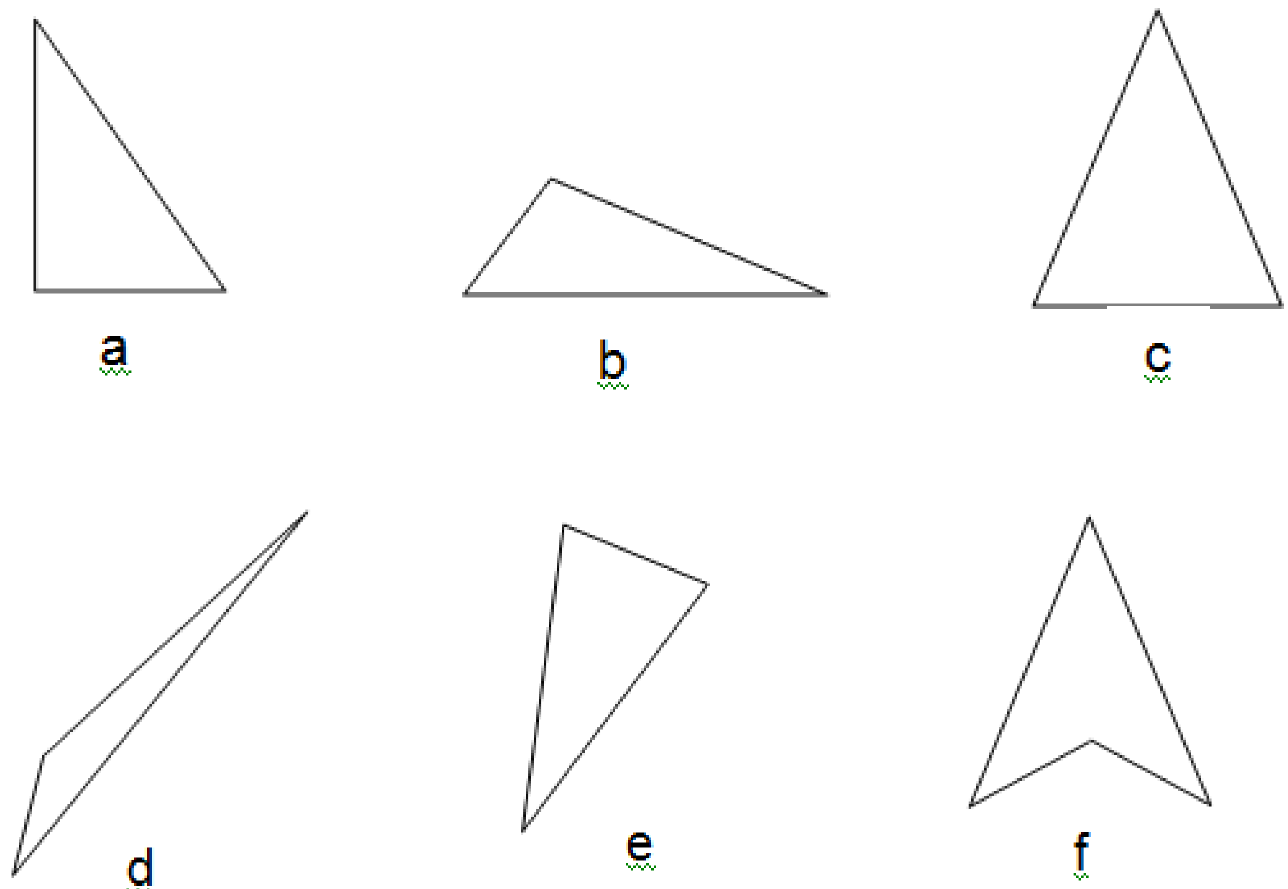
Draw a triangle

… and another

… and another

… and another

…


a
b
c
d
e
f

Conceptual and procedural variation

- Conceptual Variation
  - Positive variation
    - Standard variation
    - Non-standard variation
  - Negative variation

Conceptual: Negative variation

Solve the equation: $x^2 - 5x + 6 = 2$

Ans: $(x-2)(x-3) = 2$

$\therefore x-2=1, x-3=2$ **or** $x-2=2, x-3=1$

$\therefore x_1=3, x_2=5$ **or** $x_1=4, x_2=4$

Do you agree?

Concept and non-concept
Discerning the essence of concepts

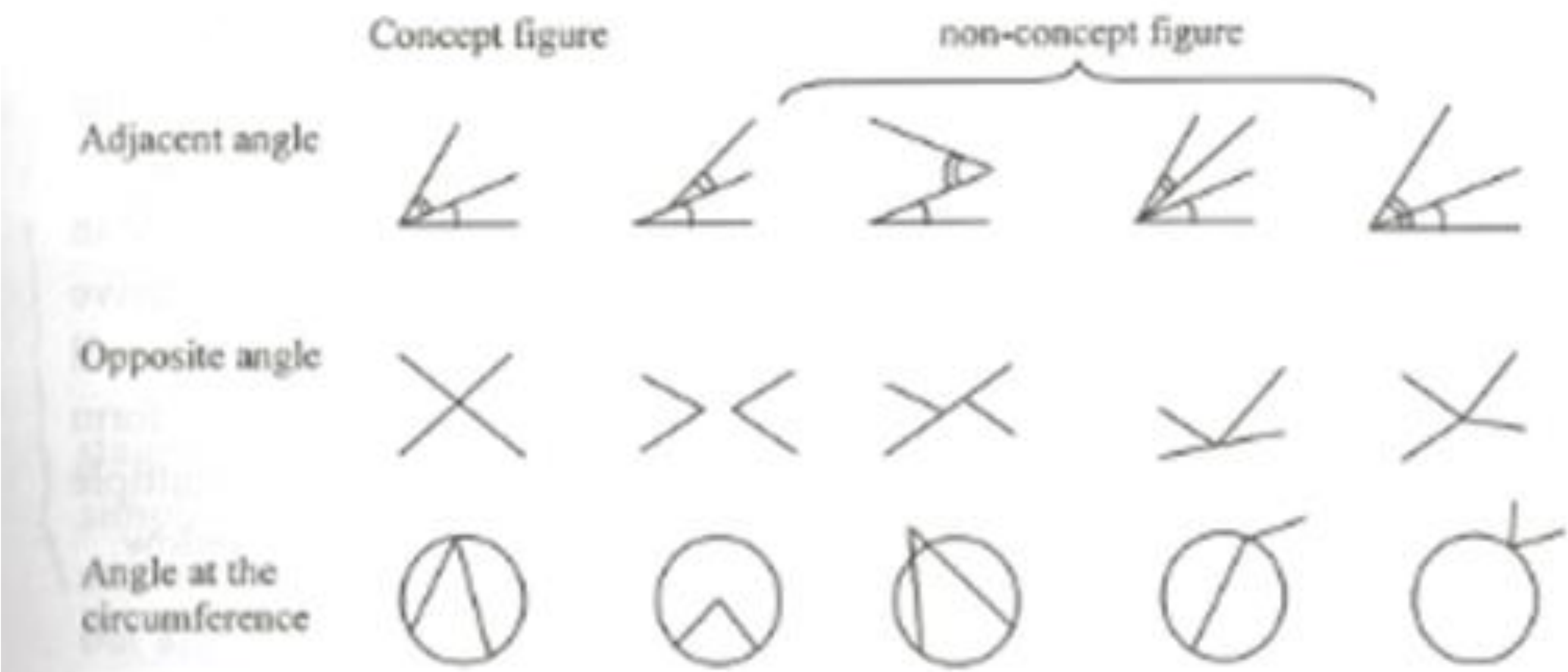


Figure 4. Non-concept figure variations for discerning the essence of concepts (L. Gu, 1981)

Conceptual and procedural variation

- Conceptual Variation
  - Positive variation
    - Standard variation
    - Non-standard variation
  - Negative variation
- Procedural Variation
  - Variation to scaffold mathematical thinking
  - Variation as a support for problem solving

Procedural Variation

Provides the opportunity

- for practice (intelligent rather than mechanical);
- to focus on relationships, not just the procedure;
- to make connections between problems;
- to use one problem to work out the next;
- to create other examples of their own.

Variation Theory in Practice

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What do you want students to attend to?

What is the key point?

Procedural variation

Write the two missing values to make these equivalent fractions correct.

$$\frac{\square}{3} = \frac{8}{12} = \frac{4}{\square}$$

Write the three missing digits to make this **addition** correct.

$$\begin{array}{r} 1\;5\;\square \\ +\;4\;\square\;4 \\ \hline \square\;1\;5 \end{array}$$

Write the missing number.

$$70 \div \square = 3.5$$

2016 KS2 National Curriculum Tests
Questions from Reasoning Papers

What is kept the same as well as what is varied.

$$x^2 - 5x + 6 = 0$$

$$x^2 - 5x - 6 = 0$$

$$x^2 + 5x - 6 = 0$$

$$-x^2 - 5x + 6 = 0$$

Conceptual and procedural variation

- Conceptual Variation
  - Positive variation
    - Standard variation
    - Non-standard variation
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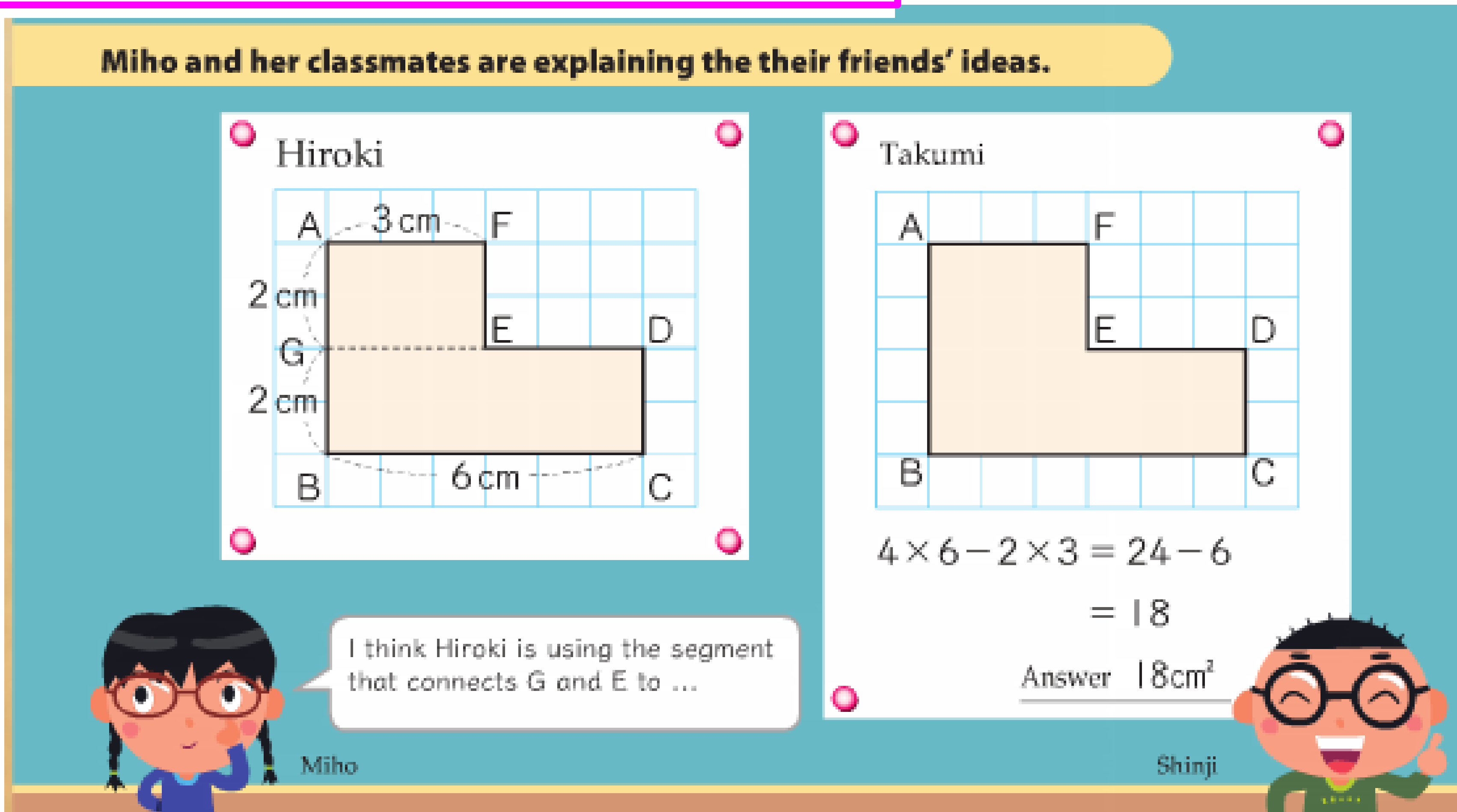
Look at what Hiroki drew and write down his ideas using math sentences.

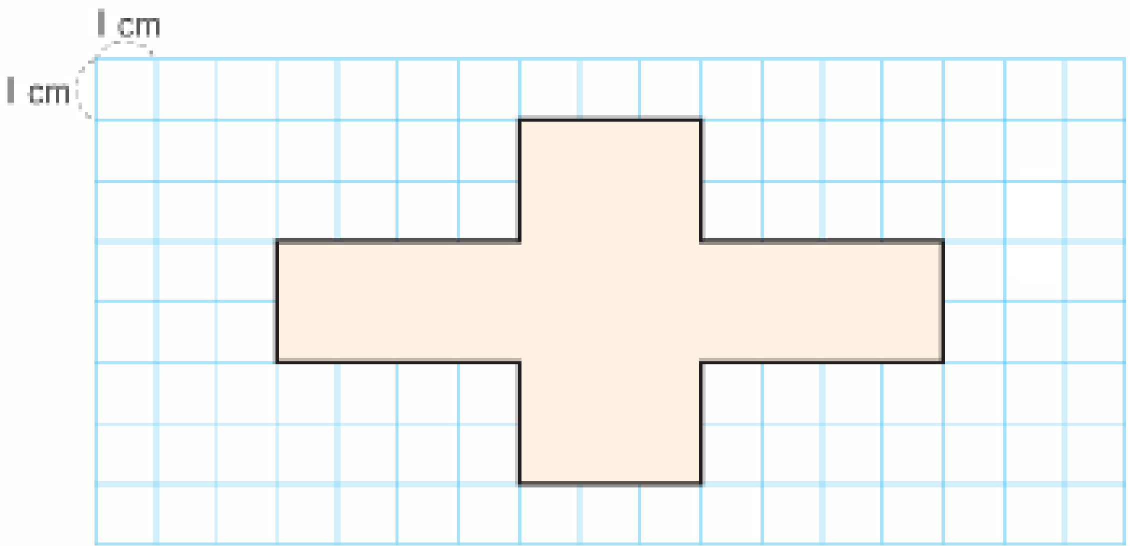


Look at the math sentence Takumi wrote and explain how he thought about the problem.

Providing Textbook Supports for Teaching Math **Akihiko Takahashi**
https://prezi.com/s1nvam1gllv9/providing-textbook-supports-for-teaching-math/

Different methods

6 Calculate the area of the shape below in many different ways.



Additional Problems → page 118 D

Let's check.

Try different problems using what you've learned today.

Purpose of variation

- Supports deep learning by providing rich experience rather than superficial contact
- Provides the necessary consolidation (in familiar and unfamiliar situations) to embed and sustain learning
- Focuses on conceptual relationships and make connections between ideas
- Supports pupils' ability to reason and to generalise

Key Ideas

1. The central idea of teaching with variation is to highlight the essential features of a concept or idea through varying the non-essential features.
2. When giving examples of a mathematical concept, it is useful to add variation to emphasise:

a. What it is (as varied as possible);
b. What it is not.

1. When constructing a set of activities / questions it is important to consider what connects the examples; what mathematical structures are being highlighted?
2. Variation is not the same as variety – careful attention needs to be paid to what aspects are being varied (and what is not being varied) and for what purpose.

Five big ideas (at primary)

Secondary Teaching for Mastery Specialists analysing and exploring these